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ABSTRACT

This document is designed to assist teachers and other school personnel in the planning and teaching of the sixth grade mathematics course. Contents include: (1) Overview of Grade 6 Mathematics (mission statement, purpose and philosophy, goals, National Council of Teachers of Mathematics' Professional Standards for Teaching Mathematics, and uses of technology and manipulatives); (2) Essential Elements of Instruction and Learning Objectives; (3) Texas Assessment of Academic Skills (TAAS) (focus, domains, objectives, targets, and instructional strategies); (4) Sample Lessons for Teaching Grade 6 Mathematics; (5) Evaluation (philosophy, types of evaluation, samples, test-taking strategies, grading, and homework); and (6) The 12 Most Important Things You Can Do to Be a Better Math Teacher. TAAS features three domains: concepts, operations, and problem solving. The Essential Elements are: problem solving; patterns, relations, and functions; number and numeration concepts; operations and computation; measurement; geometry; and probability, statistics, and graphing. Suggested resources include children's trade books, software, and suggested manipulatives. Contains 22 resources and 10 references. (MKR)

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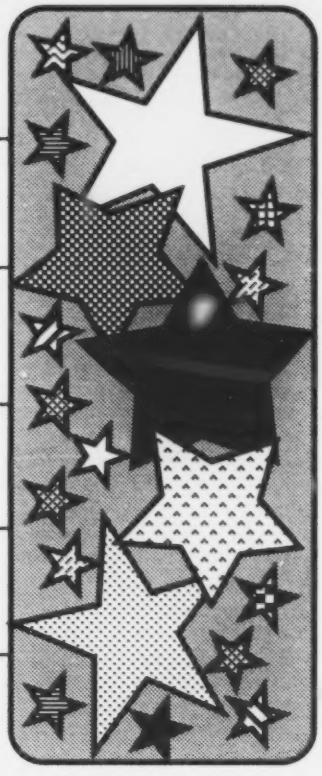

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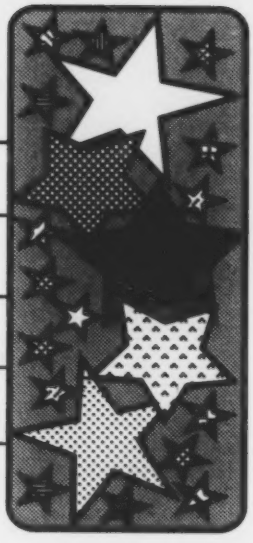

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FOREWORD

Guidelines for Teaching Grade 6 Mathematics is designed to help teachers and other school district personnel plan and teach sixth grade mathematics. The publication presents the philosophy and intent of the course and discusses the required essential elements, TAAS instructional targets, instructional strategies, and the use of technology and manipulatives. Also included are sample objectives and activities to illustrate how the essential elements for sixth grade mathematics can be taught. School district personnel may want to use these suggestions to develop their own curriculum documents for the course.

We hope these guidelines will be useful in planning and teaching mathematics for Grade 6 and in equipping the mathematics classroom.

Lionel R. Meno
Commissioner of Education

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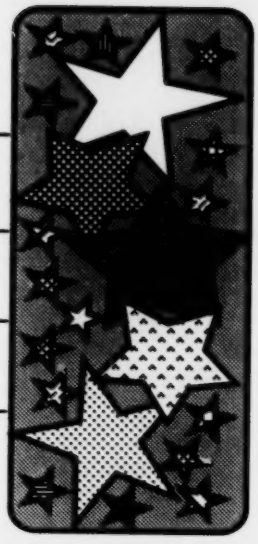
Overview of Grade 6 Mathematics



Mission Statement

Guidelines for Teaching Grade 6 Mathematics is designed to assist teachers and other school personnel in the planning and teaching of the sixth grade mathematics course. This document discusses the philosophy and intent of the course, the goals, essential elements, and sample objectives, activities, and lesson plans. Also included is a discussion of the uses of technology and manipulatives, as well as Texas Assessment of Academic Skills (TAAS) domains, objectives, and targets; evaluation techniques and samples; and suggested resources and references. School personnel may want to use these suggestions to develop their own curriculum documents for sixth grade mathematics. These guidelines will be useful to district personnel in planning curriculum, teaching mathematics, and equipping classrooms.

When students are allowed to experience the numerous perspectives of mathematics, they will be more successful. This document has been developed with that in mind. A teacher's understanding of the content matter taught is most important. A teacher's understanding of how a student can achieve mastery of concepts; how the mistakes made by students and a teacher's readiness to reduce or eliminate these occurrences; and how a teacher uses books, resources, and materials sets the stage for successful experiences in learning and teaching mathematics.

The sixth grade essential elements and the seventh grade TAAS objectives were used as a basis for the lesson activities in these guidelines. Some of the activities may be challenging for students. Challenging or advanced topics should be taught because they demonstrate to students what they are capable of learning, understanding, and accomplishing in mathematics. This document is designed to inspire teachers to inspire students to want to learn mathematics. Only when educators begin to expand their horizons in this way will a significant impact be made upon students.

Purpose and Philosophy

Mathematics is useful, exciting, and creative and can be enjoyed by all middle school students. Problem-solving skills and logical reasoning are developed while students explore and make sense of their world through the study of mathematics. Unfortunately, mathematics has been viewed by many students as boring, irrelevant, and routine and as externally dictated by a rigid system of rules governed by standards of speed, accuracy, and memory. In the past, computational facility has been emphasized instead of a broad, integrated view of mathematics. While computational skills are important, learner characteristics and the vitality of mathematics itself cannot be overlooked. Mathematics in Grade 6 should be broad based and concept driven and should reflect relevant mathematics and its interrelationships with technology.

Middle school students are in a transitional period, forming lifelong values and skills. Decisions about what students will study and how they will learn can dramatically affect their futures, and failure to study mathematics can result in a loss of opportunities. Attitudes that affect the decisions are often formed during middle school years, so the curriculum must be useful, interesting, and relevant, and it must foster a positive disposition toward mathematics.

The middle school mathematics curriculum should expand students' knowledge of numbers, computation, estimation, measurement, geometry, statistics, probability, patterns and functions, and the fundamental concepts of algebra. Traditionally, textbooks have offered the same topics, approach, and presentation grade after grade, with little change from Grade 5 through Grade 8. Chapters on new material have been included in the last half of textbooks in sections not covered because of lack of time. Thus the result has been a reexamination of materials students have already seen, which promotes a negative image of mathematics and an inadequate background for secondary school mathematics.

A broad-based and flexible view of the middle school curriculum is important for several reasons. First of all, basic skills for the 1990s and beyond mean far more than computational facility. With calculators readily available, the need for tedious paper and pencil proficiency is obsolete, and topics such as geometry, probability, and statistics have become more important through technology. Secondly, if students have been unable to master basic computational skills in elementary school, they are likely to be unsuccessful with the same techniques during the middle school years. Thirdly, many mathematical topics currently omitted actually can assist students in arithmetic concepts and skills through a fresh approach. The sixth grade curriculum should include the following:

- problem situations that establish the need for new ideas and motivate students
- communications with and about mathematics
- mathematical reasoning
- a broad range of topics, including number concepts, computation, estimation, functions, algebra, statistics, probability, geometry, and measurement
- topics taught as an integrated whole
- technology, including calculators, computers, and videos

Goals

According to the *Curriculum and Evaluation Standards for School Mathematics* (the *Standards*) developed by the National Council of Teachers of Mathematics (NCTM), the five overall curriculum goals for students are:

- learning to value mathematics
- becoming confident in their ability to do mathematics
- becoming mathematical problem solvers
- learning to communicate mathematically
- learning to reason mathematically

Moreover, the educational system of today demands new societal goals for education:

- mathematically literate workers
- lifelong learning
- opportunity for all
- an informed electorate

Specifically, teaching the mathematics curriculum to middle school students must be related to the characteristics of the learners and their needs today and in the future. *Everybody Counts* (National Research Council, 1989) posits that "self-confidence built on success is the most important objective of the mathematics curriculum" (page 45). Individuals must be able to cope with mathematics in their later lives—as employees, parents, and citizens. Ability to do so depends on attitudes toward mathematics conveyed in school. The mathematics curricula must not leave a legacy of misunderstandings and apprehension.

National Council of Teachers of Mathematics: Professional Standards for Teaching Mathematics

The *Professional Standards for Teaching Mathematics (NCTM, 1991)* are based on four assumptions about the practice of teaching. These assumptions are abbreviated versions of the more extensive ones found in the original document (NCTM, 1991, pages 21-22).

(1) The goal of teaching mathematics is to help all students develop mathematical power. Teachers must help every student develop conceptual and procedural understandings of number, operations, geometry, measurement, statistics, probability, functions, and algebra and the connections among ideas. They must engage all students in formulating and solving a wide variety of problems, making conjectures and constructing arguments, validating solutions, and evaluating the reasonableness of mathematical claims.

(2) What students learn is fundamentally connected with how they learn it. Students' opportunities to learn mathematics are a function of the setting and the kinds of tasks and discourse in which they participate.

(3) All students can learn to think mathematically. The goals such as learning to make conjectures, to argue about mathematics using mathematical evidence, to formulate and solve problems, and to make sense of mathematical ideas are not just for some group thought to be "bright" or "mathematically able."

(4) Teaching is a complex practice and hence not reducible to recipes or prescriptions. First of all, teaching mathematics draws on knowledge from several domains: knowledge of mathematics, of diverse learners, of how students learn mathematics, of the contexts of the classroom, school, and society. Good teaching depends on a host of considerations and understandings. Good teaching demands that teachers reason about pedagogy in professionally defensible ways within particular contexts of their own work.

The *Professional Standards for Teaching Mathematics* identifies a particular set of instructional standards for the effective teaching of mathematics. These standards describe the nature of the tasks, patterns of communication and the learning environment. More specifically, five of these standards focus on instructional strategies. They are:

STANDARD 1: WORTHWHILE MATHEMATICAL TASKS

The teacher of mathematics should pose tasks that are based on:

- sound and significant mathematics;
- knowledge of students' understandings, interests, and experiences;
- knowledge of the range of ways that diverse students learn mathematics;

and that

- engage students' interests;
- develop students' mathematical understandings and skills;
- stimulate students to make connections and develop a coherent framework for mathematical ideas;

- call for problem formulation, problem solving, and mathematical reasoning;
- promote communication about mathematics;
- represent mathematics as an ongoing human activity;
- display sensitivity to, and draw on, students' diverse background experiences and dispositions;
- promote the development of all students' dispositions to do mathematics.

STANDARD 2: THE TEACHER'S ROLE IN DISCOURSE

The teacher of mathematics should orchestrate discourse by:

- posing questions and tasks that elicit, engage, and challenge each student's thinking ability;
- listening carefully to students' ideas;
- asking students to clarify and justify their ideas orally and in writing;
- deciding what to pursue in depth from among the ideas that students bring up during a discussion;
- deciding when and how to attach mathematical notation and language to students' ideas;
- deciding when to provide information, when to clarify an issue, when to model, when to lead, and when to let a student struggle with a difficulty;
- monitoring students' participation in discussions and deciding when and how to encourage each student to participate.

STANDARD 3: STUDENTS' ROLE IN DISCOURSE

The teacher of mathematics should promote classroom discourse in which students:

- listen to, respond to, and question the teacher and one another;
- use a variety of tools to reason, make connections, solve problems, and communicate;
- initiate problems and questions;
- make conjectures and present solutions;
- explore examples and counterexamples to investigate a conjecture;
- try to convince themselves and one another of the validity of particular representations, solutions, conjectures, and answers;
- rely on mathematical evidence and argument to determine validity.

STANDARD 4: TOOLS FOR ENHANCING DISCOURSE

The teacher of mathematics in order to enhance discourse, should encourage and accept the use of:

- computers, calculators, and other technology;
- concrete materials used as models;
- pictures, diagrams, tables, and graphs;
- invented and conventional terms and symbols;
- metaphors, analogies, and stories;
- written hypotheses, explanations, and arguments;
- oral presentations and dramatizations.

STANDARD 5: LEARNING ENVIRONMENT

The teacher of mathematics should create a learning environment that fosters the development of each student's mathematical power by:

- providing and structuring the time necessary to explore sound mathematics and grapple with significant ideas and problems;
- using the physical space and materials in ways that facilitate students' learning of mathematics;
- providing a context that encourages the development of mathematical skill and proficiency;
- respecting and valuing students' ideas, ways of thinking, and mathematical dispositions;

and by consistently expecting and encouraging students to:

- work independently or collaboratively to make sense of mathematics;
- take intellectual risks by raising questions and formulating conjectures;
- display a sense of mathematical competence by validating and supporting ideas with mathematical argument.

Uses of Technology and Manipulatives

Calculators and computers are tapped for important roles in mathematics education at all levels and across topics. Changes in technology and the broadening of the areas in which mathematics is applied have resulted in growth and changes in the discipline of mathematics itself. The new technology has altered the very nature of the problems important to mathematics and the methods mathematicians use to investigate them.

The *Standards* call for the following regarding technology in the classroom:

- appropriate calculators for all students at all times
- a computer for every classroom for demonstration
- access to a computer for individual and group work
- students learning to use the computer as a tool for processing information and performing calculations to solve problems

Calculators and computers offer teachers and students a rich learning aid. Their potential is great and as yet untapped both in developing concepts and in developing positive attitudes and persistence in problem solving.

Computers can be utilized in a variety of ways in the mathematics classroom, and the appropriateness of a particular approach depends on the goals. Three qualitatively different methods suggested by R. Taylor in *The Computer in the School : Tutor, Tool, Tutee* are:

- as a sophisticated teaching machine
- to be programmed (or taught) by the student
- as a mode for applications in research and development through software that displays graphs, manipulates symbols, analyzes data, and performs mathematical procedures. Applications such as spreadsheets, word processing, data bases, and communication packages have the appeal of matching the classroom's use of technology with that of society's.

Calculator use is not for the purpose of replacing paper-and-pencil computations but to reinforce them. According to N. Kober in *What We Know About Mathematics Teaching and Learning*, calculator use is apt to sustain independent thought, not replace it. For example, students can be challenged to invent calculator algorithms to replace procedures taught in textbooks. The students explain why their procedures work and debate the advantages and disadvantages of their procedures over others. Calculators are programmable, produce graphics, and work in fractional and algebraic notation. Teachers need to be innovative, experiment, and share ideas.

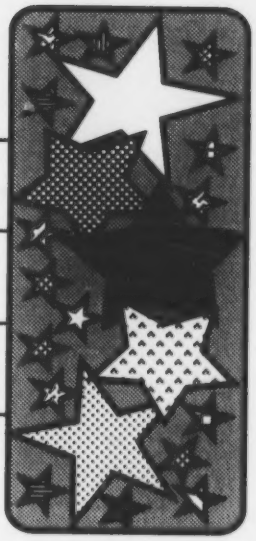
Furthermore, manipulatives offer an excellent way to enable students to connect between mathematical ideas. Learning is enhanced when students are exposed to a concept in a variety of manipulative contexts. As an example, fractions represented with colored cubes, pattern blocks, fraction bars, fraction circles, and Cuisenaire rods help students understand the concept of fractions independent of the physical representation. In addition to using manipulatives for new concepts, activities should be oriented to help students connect between concrete, pictorial, and abstract representations of ideas.

The primary goal in teaching mathematics is to develop conceptual understanding of mathematical ideas. Technology and manipulatives are tools to achieve this goal if implemented in classroom activities.

I hear and I forget. I see and I remember. I do and I understand.

Chinese proverb

Essential Elements of Instruction and Learning Objectives



Essential Elements of Grade 6 Mathematics

The State Board of Education in 1989 revised the essential elements of instruction for mathematics, Grades 1-8. These revised essential elements follow closely the recommendations made by the National Council of Teachers of Mathematics in its nationally recognized *Curriculum and Evaluation Standards for School Mathematics*. According to the Texas Education Agency (1989), "The mathematics curriculum review committee and the Agency [TEA] have tried to be sensitive to a balance between changes expected of teachers and improvements necessary to help students learn mathematics more effectively." Some of these major changes include:

- narrowing the spiral of the curriculum—beginning some topics later and finishing some topics sooner in the curriculum to eliminate some of the redundance
- revising the role of review in the curriculum so that the majority of each grade level is new material and so that review is placed in relevant contexts
- emphasizing the development of problem-solving skills in relevant and interesting situations
- incorporating calculators and computers throughout all grades as problem-solving tools
- adding an essential element on patterns, relations, and functions
- separating the teaching of operations and computation so that all students learn the meaning of the operations
- strengthening the areas of probability, statistics, and geometry

- emphasizing the importance of communication in mathematics
- building on a sound foundation of concepts rather than on rote procedures
- putting mathematics into meaningful contexts

The revised essential elements for Grade 6 are as follows:

(1) **Problem Solving.** Experience in solving problems designed to systematically develop students' problem-solving abilities through a variety of strategies and approaches. The student shall be provided opportunities to engage in the following types of activities:

(A) develop an organized approach to solving application and nonroutine problems appropriate for Grade 6;

(B) analyze problems by identifying relationships, discriminating relevant from irrelevant information, sequencing, observing patterns, prioritizing, and questioning;

(C) communicate an understanding of a problem by describing and discussing the problem and recording the relevant information;

(D) select appropriate strategies from a variety of approaches;

(E) select appropriate materials and methods for solutions; and

(F) generate and extend problems.

(2) **Patterns, Relations, and Functions.** Use of models and patterns to develop the algebraic concepts of relations and functions. The student shall be provided opportunities to:

(A) explore other numeration systems;

(B) explore patterns of exponents;

(C) build simple functions using concrete models and generate a corresponding rule; and

(D) write expressions for word phrases.

(3) **Number and Numeration Concepts.** Concepts and skills associated with the understanding of numbers and the place value system. The student shall be provided opportunities to:

(A) demonstrate the meaning of percent with concrete models;

(B) explore relationships between whole numbers, fractions, decimals, and percents;

(C) identify the appropriate equivalent form of a number (fraction, decimal, percent) in application problems;

(D) develop the concept of ratio in application problems;

(E) compare and order positive rational numbers;

(F) develop the meaning of integers as representations in problem situations;

(G) write the prime factorization for a number using exponents; and

(H) use prime factorization to investigate common factors and common multiples.

(4) **Operations and Computation.** Use of manipulatives to develop the concepts of basic operations on numbers and to apply these concepts to the computational algorithms. The student shall be provided opportunities to:

(A) select an appropriate operation and/or strategy to solve a problem and justify the selection;

(B) use the order of operations to solve multi-step problems using a calculator when appropriate;

(C) divide whole numbers resulting from problem situations;

(D) multiply and divide decimals in application problems;

(E) add and subtract fractions in problem situations;

(F) multiply and divide fractions using concrete models and connecting to rules;

(G) estimate and solve problems using ratios and proportions;

(H) use ratios to solve probability problems; and

(I) write and solve simple linear equations from problem situations and check the reasonableness of the result.

(5) **Measurement.** Concepts and skills using metric and customary units. The student shall be provided opportunities to:

(A) determine the degree of accuracy required in measurement for a specified purpose;

(B) approximate the area of irregular figures using grids;

(C) develop and apply area formulas to common polygons and circles;

(D) estimate answers and solve application and nonroutine problems involving area;

(E) explore the effect on area when a dimension of a two-dimensional figure is changed;

(F) use models to develop and apply the formula for the volume of rectangular solids;

(G) use the relationship between units to convert measures within the same measurement system;

(H) use denominate numbers to solve application problems; and

(I) measure and compare angles.

(6) **Geometry.** Properties and relationships of geometric shapes and their applications. The student shall be provided opportunities to:

(A) identify and distinguish between similar, congruent, and symmetric figures;

(B) visualize and sketch the results of a rotation, translation, or reflection using graphing technology when appropriate;

(C) build models of three-dimensional figures such as pyramids, cones, or prisms with polygonal bases and investigate the properties associated with those figures;

(D) construct angles and angle bisectors;

(E) classify angles and pairs of lines, including skew lines;

(F) classify polygons by sides and angles; and

(G) recognize the application of geometry in such areas as nature, art, architecture, construction.

(7) **Probability, Statistics, and Graphing.** Use of probability and statistics to collect and interpret data. The student shall be provided opportunities to:

(A) collect, organize, and interpret data to solve application problems;

(B) construct and interpret circle graphs using calculators and computers when appropriate;

(C) compare quantities using ratios;

(D) determine the extent to which the results of a sample population can be generalized to a larger population;

(E) determine and interpret fractional probabilities of simple events;

(F) compare theoretical probabilities and experimental probabilities of an experiment; and

(G) plot points on a coordinate plane that represent ordered pairs of whole numbers, simple fractions, or decimals, arising from application problems.

Sample Learning Objectives

The following are sample objectives that mathematics educators could use to teach the essential elements for Grade 6.

ESSENTIAL ELEMENT 1: Problem Solving

(A) Use a problem-solving heuristic (Understand, Plan, Solve, and Look Back) to solve application and nonroutine problems.

(B) Recognize problems having superfluous or insufficient information. Order the events in a problem. Describe patterns observed in problems.

(C) Identify the question asked in a problem. Rewrite a word problem to describe the question, facts, and key ideas.

(D) Select and use a variety of problem-solving strategies (e.g., guess and check, find a pattern, make a list, use a model, make a table, eliminate, simplify, choose an operation, work backwards) to solve word problems.

(E) Use concrete models (base ten blocks, color tiles, pattern blocks) to solve problems.

(F) Write word problems for each of the problem-solving strategies listed in Objective (D) above.

ESSENTIAL ELEMENT 2: Patterns, Relations, and Functions

(A) Use base five blocks to show arithmetic operations and place value concepts.

(B) Use concrete models to explore patterns with exponents.

(C) Use concrete models to show a simple function then write a rule for the relationship.

(D) Write algebraic expressions for word phrases.

ESSENTIAL ELEMENT 3: Number and Numeration Concepts

(A) Use 100-square grids (percent grids) to illustrate percents.

(B) Use concrete models to show the relationships of the arithmetic operations of addition, subtraction, multiplication, and divison to whole numbers, fractions, decimals, and percents.

(C) Identify and use fractions, decimals, and percents in application problems.

(D) Use ratios to solve application problems.

(E) Use concrete models (such as fraction bars, decimal squares, percent grids) to show the relationships of equality, inequality, and order between fractions, decimals, and percents.

(F) Use two-color chips or numberlines to represent integers in problem-solving situations.

(G) Use exponents and prime numbers to factor a number.

(H) Use the prime factorization of two or more numbers to find common factors and multiples.

ESSENTIAL ELEMENT 4: Operations and Computation

(A) Given a mathematical word problem, identify an appropriate operation(s) and problem-solving strategy needed to solve it; justify the selection.

(B) Use a calculator and the order of operations to solve multi-step problems.

(C) Solve word problems using division.

(D) Solve application problems using multiplication and division of decimals.

(E) Solve problem situations using addition and subtraction of fractions.

(F) Use concrete models (such as fraction bars) to multiply and divide fractions.

(G) Use ratios, proportions, and estimation to solve problems.

(H) Solve probability problems using ratios.

(I) Solve problem situations using simple linear equations. Check the reasonableness of each result.

ESSENTIAL ELEMENT 5: Measurement

(A) Decide on the degree of accuracy of measurement required in a problem-solving situation.

(B) Use grids to determine the approximate area of irregular figures (such as a hand print or foot print).

(C) Use concrete objects (such as color tiles, pattern blocks, geoboards) to develop the formula for area of common polygons and circles.

(D) Use estimation strategies to estimate answers and to solve application and nonroutine problems involving area.

(E) Change one dimension of a two-dimensional figure and explore the effect of that change on the area of the figure.

(F) Develop the formula for the volume of rectangular solids by using concrete models (such as filling containers, boxes, etc.), then apply that formula to other problem-solving situations.

(G) Convert measures within the same measurement system.

(H) Solve application problems using denominate numbers.

(I) Use tracing paper, compasses, and protractors to measure and compare angles.

ESSENTIAL ELEMENT 6: Geometry

(A) Identify figures that are similar, congruent, and symmetric.

(B) Visualize, sketch, and graph the rotation, reflection, and translation of a figure.

(C) Build models of pyramids, cones, and prisms with polygonal bases and identify properties of these figures.

(D) Use a compass and straight edge to construct angles and angle bisectors.

(E) Classify angles and pairs of lines (including skew lines).

(F) Classify polygons by sides and angles.

(G) Apply geometric ideas to areas such as art, architecture, and construction.

ESSENTIAL ELEMENT 7: Probability, Statistics, and Graphing

(A) Solve application problems by collecting, organizing, and interpreting data sets.

(B) Construct circle graphs using a data set.

(C) Use ratios to compare two or more quantities.

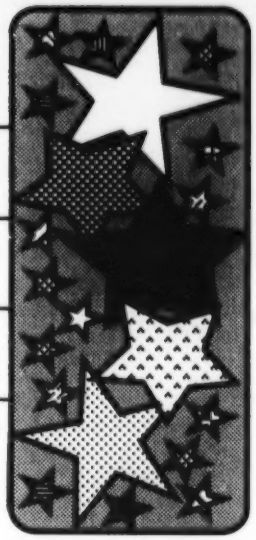
(D) Determine whether the results of a sample population can be generalized to a larger population.

(E) Determine and interpret fractional probabilities of simple events.

(F) Conduct experiments involving the probabilities of an event(s) and compare experimental results with the theoretical probabilities of the event(s).

(G) Graph points of ordered pairs of whole numbers, fractions, or decimals to solve application problems.

Texas Assessment of Academic Skills



Focus

The Texas Education Agency implemented the Texas Assessment of Academic Skills (TAAS) testing program in 1990. The program is in effect for the 1990-1995 period. The purpose of the assessment program is to provide Texas schools with an accurate measure of student achievement. The scope of content of the TAAS includes more of the instructional targets delineated in the essential elements than previous state assessments. Every section of the TAAS test contains a certain number of broad objectives. These objectives remain constant from grade to grade because they represent the core concepts that form the basis for a sound instructional progression from Grade 1 through Grade 12. What will differ from grade to grade are the instructional targets—or essential elements that comprise each objective. A portion of this extended set of instructional targets is selected for assessment annually, but not every target is tested every year.

The broadened scope of the TAAS assessment program allows for a different focus, one that addresses the academic requirements of the 1990s. Skill areas that demand little more than rote memorization are de-emphasized, while areas that improve a student's abilty to think independently, read critically, write clearly, and solve problems logically receive increased emphasis. This emphasis is in keeping with current national trends in education, which stress the importance and necessity of teaching students higher order thinking skills.

Domains, Objectives, and Targets

The TAAS features three domains—concepts, operations, and problem solving. Each domain contains objectives that are derived from the essential elements. For every objective, there are instructional targets that describe the kinds of mathematical experiences that will reflect that objective. Each instructional target was taken for the most part directly from the essential elements as delineated in the *State Board of Education Rules for Curriculum*. Each target is defined in behavioral terms appropriate for pencil-and-paper testing.

DOMAIN: Concepts

Objective 1: The student will demonstrate an understanding of number concepts.

(a) Compare and order nonnegative rational numbers, excluding whole numbers
(b) Round whole numbers and decimals (to nearest tenth, one, ten, or hundred)
(c) Determine relationships between and among fractions (denominators of 2, 3, 4, 5, 6, 8, and 10), decimals, and percents
(d) Use exponential notation to represent whole number expressions
(e) Factor whole numbers
(f) Find the least common multiple and the greatest common factor

Objective 2: The student will demonstrate an understanding of mathematical relations, functions, and other algebraic concepts.

(a) Use nonnegative rational number properties and inverse operations
(b) Determine missing elements in patterns
(c) Find relationships between ratios
(d) Solve simple linear equations
(e) Identify ordered pairs on a coordinate plane
(f) Use number line representations of fractions and decimals

Objective 3: The student will demonstrate an understanding of geometric properties and relationships.

(a) Recognize properties of two- and three-dimensional figures
(b) Identify translations, reflections, rotations, and their applications
(c) Recognize similarity, congruence, and symmetry

Objective 4: The student will demonstrate an understanding of measurement concepts using metric and customary units.

(a) Use metric and customary units
(b) Convert within the metric system
(c) Convert within the customary system
(d) Find perimeter and circumference
(e) Determine area (with and without grids) and volume

Objective 5: The student will demonstrate an understanding of probability and statistics.

(a) Use counting arrangements
(b) Use sample spaces to find fractional probability
(c) Predict possible outcomes from a sample
(d) Analyze data and interpret graphs
(e) Find means (averages)

DOMAIN: Operations

Objective 6: The student will use the operation of addition to solve problems.

(a) Add whole numbers, fractions, and decimals

Objective 7: The student will use the operation of subtraction to solve problems.

(a) Subtract whole numbers, fractions, and decimals

Objective 8: The student will use the operation of multiplication to solve problems.

(a) Multiply whole numbers and decimals

Objective 9: The student will use the operation of division to solve problems.

(a) Divide whole numbers and decimals

DOMAIN: Problem Solving

Objective 10: The student will estimate solutions to a problem situation.

(a) Estimate with whole numbers and decimals

Objective 11: The student will determine solution strategies and will analyze or solve problems.

(a) Formulate strategies or solve problems using basic operations with whole numbers, fractions, and decimals
(b) Determine strategies for solving or solve problems requiring the use of geometric concepts
(c) Analyze or solve problems through the use of similarity, congruence, and symmetry
(d) Analyze or solve problems using probability and statistics concepts

Objective 12: The student will express or solve problems using mathematical representaiton.

(a) Formulate solution sentences
(b) Analyze or interpret graphs and charts and use the information derived to solve problems

Objective 13: The student will evaluate the reasonableness of a solution to a problem situation.

(a) Evaluate reasonableness

Instructional Strategies

A 50-Minute Lesson Format

1. Eye Opener Activity/Magic Tricks—3 minutes
2. Vocabulary and Facts—10 minutes
3. Main Lesson—30 minutes
4. Mental Math—5 minutes
5. Closure/Review—2 minutes

Eye Opener:
In the school library are magic books. Finding a trick that may or may not be math related will enhance your students' interests. Students will look forward to the eye opener. The eye opener may also be some type of activity that is quick and to the point. This may be teaching the students how to multiply by nine using their fingers.

Vocabulary and Facts:
This is essential for the students. Ask students at the beginning what is the product of 2 and 3? Most will probably not know if they have not previously studied math vocabulary. Note however that most all the students recognize the number form of 2 x 3 = .

Main Lesson:
Having done the above, students are ready for the main lesson.

Mental Math:
End the math class by teaching students a mental math skill. For example, how to multiply by 25 mentally.

Closure/Review:
Summarize the lesson's activities and concepts.

Following Directions

Administer this test to the students at the beginning of the school year to stress the importance of following directions. Tell students this is a timed test. They have one minute to work on it. Place the test face down on their desks. Say "start" for students to begin. Observe how many students did not read directions. You may be surprised!

DIRECTIONS:

- This is a timed test.
- Work quickly and carefully.
- Write the answers in the blanks provided.
- Work only problems three and ten.

1. 11 + 4 =
2. 6 x 3 =
3. Your name is
4. Raise your right hand!
5. 13 - 3 =
6. Stand up, then sit down!
7. 20 x 0 =
8. Walk quickly to the wall closest to you, touch the wall, then go back to your seat and sit down!
9. 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 =
10. Come to the front and turn in your paper!

Twelve Suggestions that Work

1. The eye opener activity at the beginning of every session helps to ensure that students are attentive and focused prior to the presentations of mathematics materials.

2. The facts and vocabulary activity provides students an opportunity to be successful (during reviews); to strengthen their knowledge of mathematics (facts/vocabulary); to respond openly in a group setting.

3. The quick analysis that follows each session improves presentations that follow.

4. Providing instructions/expectations prior to each activity improves the student's opportunities for success.

5. Rewarding students with praise or small gifts after exemplary performances or active participation reinforces the importance of the learning process.

6. Activities that involve each individual in the group make use of peer pressure which improves participation by students who tend to be passive learners (chanting, spelling vocabulary words, walking among students, and insisting on participation by reluctant participants proves to be motivational).

7. Treating the taking of notes as a following instructions activity helps in getting students to focus in activities.

8. Mental games (Number Sense and quick answers to problems involving a series of operations) prove challenging and a change of pace to the regular classroom routine.

9. The rapid move from the eye opener activity to the facts vocabulary activity to the main concept activity to the mental math activity to the review activity has students constantly engaged.

10. The daily handouts that are provided for student use serve to reinforce facts, vocabulary, and concepts introduced during the sessions.

11. Constantly reminding students of the importance of listening, following directions, being observant, and taking an active role in the session's different activities reinforces expectations set forth at the beginning.

12. The diagnostic test administered at the beginning of the program helps to determine group needs.

Fifty Two-Card Deck: A Teaching Tool for Basic Facts

Consider using a standard deck of playing cards to introduce or review certain facts with your students. Prior to introducing the use of playing cards, familiarize students with the following information (if possible, use an actual deck of cards).

1. Each deck consists of four suits:
   - A) black cards
     1. spades
     2. clubs
   - B) red cards
     1. hearts
     2. diamonds

   Tell students that the four suits can represent the four seasons in a year (spring, summer, fall, and winter).

2. Each suit contains 13 cards: Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King. The Jack, Queen, and King are called *face cards*. If you let the Ace = 1, Jack = 11, Queen = 12, and King = 13, the sum of all the playing cards is 364. Since the deck contains 2 Jokers, you can make the following observation: The 52 playing cards and Joker (Joker = 1), when added, equals 365 (the number of days in a year). The extra Joker (Joker = 1), when added, give 366 (the number of days in a leap year).

   The standard deck has 12 face cards (NOTE: Each suit has 3 face cards. Since there are 4 suits, there are 4 x 3 or 12 face cards). The 12 face cards can represent the number of months in one year.

3. Each deck has 52 regular playing cards and 2 Jokers. The 52 regular playing cards can represent the number of weeks in a year.

4. A year measures the time it takes the Earth to revolve once around the sun. Since it takes the Earth 365 1/4 days to revolve once around the sun, then one year = 365 1/4 days. For convenience, it was agreed to let 1 year = 365 days with the understanding that every four years, the accumulated 1/4's (1/4 + 1/4 + 1/4 + 1/4 = 1) would require a leap year consisting of 366 days.

<u>Resource</u>
McAllen ISD's *The Magic of Math* handbook for teachers.

Thinking Mathematically: Extra Time Fillers

Using Number Concepts

1. Two to four players, two sets of number cards numbered 0 to 9, paper and pencil. Draw a game board, one per player, that contains four rectangles. Shuffle the cards and place them face down. The first player takes the top card and writes the digit in any of his or her rectangles. The next player draws the next card and writes the digit in any of his or her rectangles. Continue until each player has all four digits filled. The player with the largest number wins. The game may also be played so that the winner is the player with the smallest number. Guide the students into realizing that there is a method of placing the numbers in the rectangles.

2. Draw a grid of squares, 3 by 3. Fill in the squares with the numbers 2 through 10 so that each row, column, and diagonal adds up to 18.

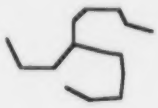
Visual Reasoning

3. Think of shapes being stretched and bent. Imagine that they are made of rubber bands. These figures are both closed loops.

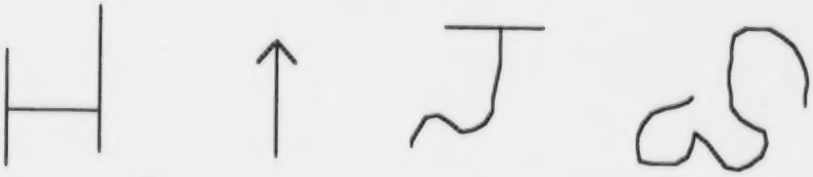
Which of these figures are equivalent to those above? (a and b) (Both are closed figures.)

a. b. c. d.

Look at this figure.



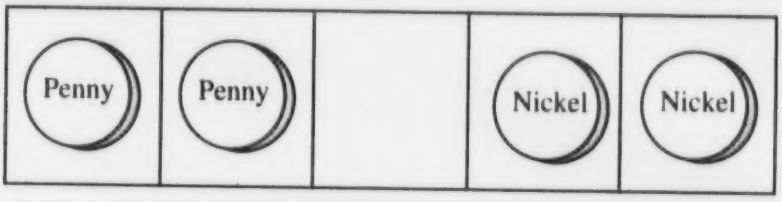
Which of the figures below could be made from it? (b and c) (Both are made up of three line segments with a common connecting point.)



b. c. d.

Investigating Patterns

4. Draw a five square board. Place two pennies on the left and two nickels on the right. The object of the game is to switch the order of the coins. Coins may move one square into an open square or jump over one coin. Pennies can move only right and nickels only may move to the left. The first move is always with a penny.



Resource

Mathematics in Action. Macmillan/McGraw-Hill, 1991, New York, NY.

Mathematics Shortcuts

Here are a few quick mathematics shortcuts which students enjoy learning and using:

When multiplying by 25, divide the other number by 4. If there are no remainders, add two 0's to the answer. If there is a remainder of 3, write a 75 at the end. If there is a remainder of 2, write a 50 at the end. (3/4 = 75%, 2/4 = 1/2 = 50%)

When multiplying by 50, divide the other number by 2. If there is no remainder, write two 0's at the end of the answer. If there is a remainder of one, then write 50 at the end instead of the two 0's.

When multiplying a two-digit number by 101, take the two-digit number and write it twice: 23 x 101 = 2323.

When multiplying a number by 100, take the decimal point of the other number and move it two times to the right.

When multiplying a number by 1000, take the decimal point of the other number and move it three times to the right.

When multiplying a number by 10000, take the decimal point of the other number and move it four times to the right.

When dividing a number by 10, take the decimal point of the other number and move it once to the left.

When dividing a number by 100, take the decimal point of the other number and move it to the left two times.

When dividing a number by 1000, take the decimal point of the other number and move it three times to the left.

Squaring two-digit numbers ending in 5:

Always write 25 for the last part of your answer. Add one to the other digit. Take this answer and multiply it to that first digit:

$\underline{35^2}$

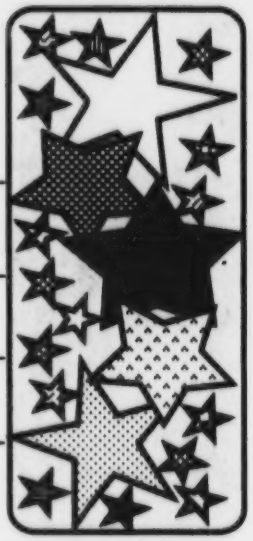
$5^2 = \underline{25}$ Always write 25 for the last part of your answer.

$4 \times 3 = \underline{12}$ Add one more to the 3, you get 4. Multiply 3 x 4 and you get 12 which is the first part of the answer.

Therefore: $\underline{35^2 = 1225}$

Always write a 25 for the last part of your answer. To get the first part, always add one more to the tens place digit and multiply these two numbers.

Sample Lessons for Teaching Grade 6 Mathematics



The following lessons represent the kind of mathematical experiences recommended for students in sixth grade mathematics classes. Several different manipulatives are included in these activities. It is important that students use these manipulatives as they work through the activities. Manipulatives and concrete objects enable middle school students to better understand the mathematical problems and concepts they so often struggle to learn. Students' experiences with manipulatives are recommended in the essential elements, TAAS's instructional targets, and NCTM's *Curriculum and Evaluation Standards for School Mathematics.*

Many activities in this section also recommend that students work together in pairs or small groups. Working together in cooperative groupings promotes communication, mathematical confidence, and students' problem-solving abilities.

EE: 1B *Related EEs: 1D, 4I* *TAAS Objectives: 2B, 11A*

Objective The learner will develop an organized approach to solving application and nonroutine problems by identifying relationships, observing patterns, and sequencing.

Activity Build physical patterns using cubes, record patterns numerically in a table, look for and use patterns to extend the table.

Materials Paper, pencil, ruler, cubes, various patterns

Resources *Algebra Thinking: First Experiences*, Creative Publications

Procedure

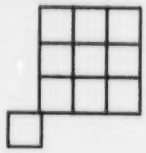
1. Supply the students with the first three examples of a pattern. Students should use cubes and build examples to match the page. They should build the fourth and fifth examples in the pattern and record them in the table. They should study the patterns and predict the entries for the tenth and one-hundredth examples.

2. The patterns can be extended three ways. The most concrete is to build each example with cubes, count, and record. Some students may notice that the list of numbers on the right is increasing by some number pattern and use this pattern to continue the list. The most advanced method is to find a general rule that relates any number in the first list to the corresponding number in the second list.

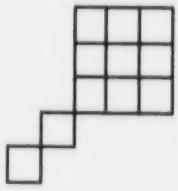
3. Students may work individually, in pairs, or in groups of three or four.

Pattern Examples

Pattern 1



A kite with 1 cube for its tail takes 10 cubes to build.



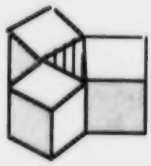
A kite with 2 cubes for its tail takes 11 cubes to build.

A kite with 3 cubes for its tail takes 12 cubes to build.

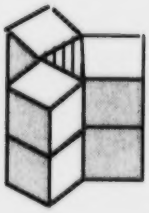
How many cubes are needed to build a kite with 10 cubes for its tail?
How many cubes are needed to build a kite with 100 cubes for its tail?

Cubes in Tail	1	2	3 • • •	10 • • •	100 • • •	n
Cubes in All	10	11	12 • • •	___ • • •	___ • • •	___

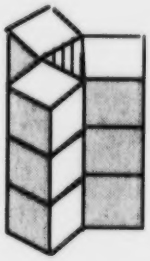
Pattern 2



A space house with 1 floor takes 3 cubes to build.



A space house with 2 floors takes 6 cubes to build.

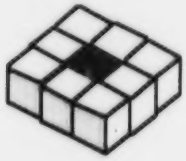


A space house with 3 floors takes 9 cubes to build.

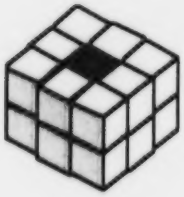
How many cubes are needed to build a space house with 10 floors?
How many cubes are needed to build a space house with 100 floors?

Number of Floors	1	2	3	• • •	10	• • •	100	• • •	n
Cubes in All	3	6	9	• • •	___	• • •	___	• • •	___

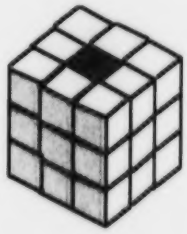
Pattern 3



A well with 1 layer takes 8 bricks to build.



A well with 2 layers takes 16 bricks to build.



A well with 3 layers takes 24 bricks to build.

How many bricks are needed to build a well with 10 layers?
How many bricks are needed to build a well with 100 layers?

Number of Layers	1	2	3	• • •	10	• • •	100	• • •	n
Bricks in All	8	16	24	• • •	___	• • •	___	• • •	___

Evaluation Completed patterns to determine the one-hundredth example. Students may also develop patterns of their own and have partners discover the patterns.

Reteach Review skip counting with the students. This will help them discover some patterns. Provide patterns of pictures. This visualization may help students find what comes next.

Extension Make the algebra connection by teaching the students to record the rule using the n notation as given in solutions.
2, 3, 8, 63, ___, __
Determine the next two numbers in the pattern and record the rule using the n notation ($n^2 - 1$).

EE: 1B *Related EEs: 1A, 1C, 1D* *TAAS Objectives: 1A, 11A*

Objective The learner will analyze problems by identifying relationships, discriminating relevant from irrelevant information.

Activity The learner will separate information given into relevant and irrelevant information.

Materials Tag board sentence strips, masking tape

Resources *The Problem Solver 6,* Creative Publications

Procedure

1. Write each of the following sentences on a separate strip of tagboard.

   Maria weighs 85 pounds. Steven weighs 110 pounds. Maria's father weighs 180 pounds. How much heavier is Maria's father than Steven?

2. Using masking tape, tape the four sentence strips on the board or wall in front of the students. Read the sentences to the class aloud. Have the students read the sentences in chorus. Have the students note that the question is asking that one compare the weights of Maria's father and Steven. Tell students that when one compares in this way one must subtract to find the difference in weights. At this point, have one of the students remove the sentence strip which is not needed to solve this problem. The sentence strip that states, "Maria weighs 85 pounds" is not necessary. Tell students that this sentence is irrelevant--not necessary to solve the problem. The sentences used are relevant--necessary to solve the problem. Have students repeat after you the words *relevant* and *irrelevant.*

3. Have students at this point work out the problem. Students should subtract 110 from 180. The correct answer is: Maria's father is 70 pounds heavier than Steven.

4. Use the same first three sentence strips for the next problem, but change the question. On another sentence strip, write the following question: How much heavier is Steven than Maria? Follow the same procedure as above in assisting students to discover relevant and irrelevant information. Have students work the problem. Students should have discovered that the sentence strip that states "Maria's father weighs 180 pounds" is irrelevant, therefore, not necessary in solving the problem. The correct answer should be "Steven is 25 pounds heavier than Maria."

Evaluation

Give students a handout containing the following sentences in the form of sentence strips that students can cut out.

> Mountain A is 10,234 feet high. Mountain B is 5,003 feet high. Ed is 5 feet tall. Sally is 4 feet tall.

Have students cut these sentence strips. Give each student a sentence strip you have made containing a question based on the information given. Examples of questions could be:

> How much higher is Mountain A than Sally? How much taller is Ed than Sally? How much smaller is Sally than Ed?

Have students separate the relevant strips from the irrelevant. Students are to work the problem at this point. Monitor students for accuracy.

Extension Students create their own word problems with relevant and irrelevant information.

EE: 1D *TAAS Objective: 12A*

Objective The learner will use guess and check as an appropriate strategy in solving word problems.

Activity Use guess and check approach to solve problems.

Materials Handout

Procedure

1. Interrelate estimation as one way of assisting students in using the guess and check approach. Give students the following problem:

   Mary would like to buy one of each of the following items in a store:
   soft drink - $0.55, candy bar - $0.45, pen - $1.35, loose-leaf paper - $1.50, book - $3.65.
   Mary has $9.00. Will she be able to get all she wants?

2. Have students each make a guess by using estimation. Have students combine items and come up with an estimated price.

   The soft drink and the candy bar will cost about $1.00. The pen and the loose-leaf paper will cost about $3.00. The book will cost about $4.00. That's a total of about $8.00. She will certainly have more than enough to buy all the items she wants.

3. Have students practice on a few more of these kinds of problems. Relate to students how useful guessing can be when they walk into a store without a calculator and want to know if they have enough money to buy what they want.

4. Introduce test-taking strategies to the students as follows:

   Give students the following problem:

   John wants to buy 40 posters at $1.55 each. About how much will the 40 posters cost?
   a. $20.00 b. $25.00 c. $222.00 d. $62.00

   By guessing, the students can figure out the most sensible choice.
   40 x $2.00 = $80.00. Therefore, $62.00 would be the closest answer.

Evaluation For evaluative purposes, have five objects you brought from home on display. Example: soap, dozen eggs, milk, 6 pack of cola, and a loaf of bread. Have the actual price of each item on display. Have students estimate (guess) the total cost of the items without the use of paper and pencil or calculator.

Reteach Review rounding numbers with students experiencing difficulty. Transfer that knowledge into use in working problems as above.

Extension Have students guess the length of selected objects. Have students guess the weight of selected objects. Have students guess the temperature inside and outside the classroom. Have them compare their guesses with the actual measurements.

Objective The learner will use acting out as a strategy for solving a word problem.

Activity Act out the problem and come to its solution.

Materials Simple props

Procedure

1. Give the students the following word problem:

   A school bus can seat 60 people. Will 7 buses be enough to take 373 students on a field trip?

2. Give one student a cut-out form of a school bus and ask the student how many students fit in the bus. The student should respond, "60."

3. Ask the students in the classroom how many more students are left. Students should respond that 300 students still need transportation. Give another student another cut-out of a bus.

4. Ask the student how many passengers will fill his or her bus. The response should be "60." That makes two buses loaded with passengers. Therefore, 120 students have transportation. Ask how many more students need transportation. The response is "120." Continue doing this until you get 6 full buses.

5. Ask students how many students are left without transportation. The response is "13." Students should, therefore, realize that a seventh bus is needed for the 13. Students will surely discover that it is easier to divide 373 by 60. Unfortunately, many students forget the remainder and forget to add the extra bus. Through acting out, students will be helped to remember the remainder.

6. As a variation of this problem, one student could represent 20 passengers. For each bus arriving, have 3 students representing 60 passengers line up behind the student with the cut-out form of the bus. Have one student represent the 13 students left.

Evaluation For evaluative purposes, you may want to divide the class into small groups of three to four students. Give each group a word problem to act out. Challenge students to be creative in their acting

Reteach Have students come up with their own word problems they will act out. Assist students as needed.

Extension Give students the following problem:

A car travels at a speed of 55 miles per hour. How far will the car travel in 5 hours?

Have students act this problem out by having a designated driver travel in their favorite car and stop every hour to record miles travelled. At the end of 5 hours, the solution to the problem would be recorded. At this point you may introduce the distance formula as a follow up: Distance = Rate x Time

EE: ID *Related EE: 41* *TAAS Objective: 12A*

Objective The learner will use models as an appropriate strategy in problem solving.

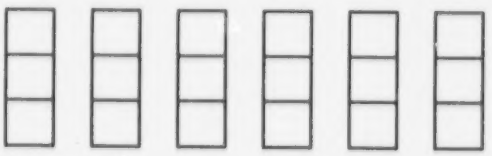
Activity Using rainbow cubes, build models to show combinations and permutations.

Materials Paper, crayons, pencil, rainbow cubes

Resources *Algebra Thinking: First Experiences*, Creative Publications

Procedure

1. Guide the students examples of permutations by using rainbow cubes and following the rules given. Builders are building three-floor towers. They can use only three colors. The three floors in each tower must be different colors. Find all the possible towers. Draw and color the possible towers.



New rule: The floors can be any combination of the three colors. Find all the towers. Draw and color the possible towers.

2. Continue by giving various other rules or extending the number of floors.

3. Have the students find the number of permutations possible for windows. Create rules and number of window panes.

4. Students may work in pairs or groups of three or four.

Evaluation Completed models of the correct permutations according to the rules given. Have the students create their own rule for permutations.

Reteach Decrease the number of floors in the tower, panes in the windows, and/or the number of colored cubes to be used. Use pictures of people and have the students find the number of ways the students could sit in the classroom.

Extension Have students write equations for the permutation rules. If the rule says there are three colors to choose from and all colors must be different, the equation is three colors possible for the first cube times two colors possible for the second cube times one color possible for the third cube:

$3 \times 2 \times 1 = 6.$

Have the students find the number of permutations possible: A bank has a safe combination that contains seven numbers. Any number from 0 to 25 can be used. No number is repeated. (Since the numbers can be 0 to 25, 26 numbers are possible for the first choice, 25 for the second, 24 for the third, and so on.)

$26 \times 25 \times 24 \times 23 \times 22 \times 21 \times 20 = 3{,}317{,}312{,}000$

EE: ID *Related EE: 41* *TAAS Objective: 12A*

Objective The learner will utilize a picture or diagram as a strategy in problem solving.

Activity Draw pictures and/or diagrams to aid in solving a problem.

Materials Paper, crayons or colored pencils, pencil, ruler

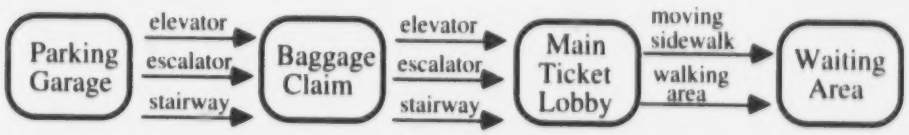
Resources *The Problem Solver*, Creative Publications

Procedure

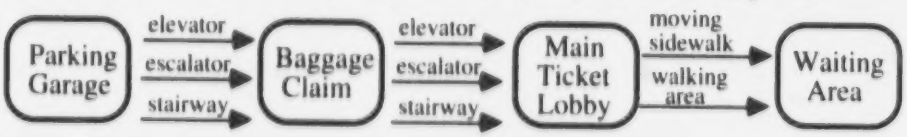
1. Guide the students through the process needed to solve the following problem:

   Mr. Puente's sixth grade class is taking a field trip to the airport. The school bus will park in the covered garage, and students will take either an elevator, escalator, or stairway to the baggage claim area. From the baggage claim area, they will take either an elevator, escalator, or a stairway to the main ticket lobby. From the lobby, they will take the moving sidewalk or walk to the waiting area for the incoming flights. How many different ways can Mr. Puente's class take from the garage to the waiting area?

2. Assist the students in drawing a picture or diagram of the airport as described in the problem.



3. Using a crayon or colored pencil, draw a path starting in the garage, running through the elevator into the baggage claim area. Continue through the second elevator and into the lobby. From the lobby, continue the line through the moving sidewalk and into the waiting area. This is one way to go from the garage to the waiting area.



4. Picking a different colored crayon or colored pencil, draw a line from the garage through the elevator into the baggage claim area. Continue through the second elevator and into the lobby. From the lobby, continue the line through the area used for walking to the waiting area.

5. For the third line, choose another color and start the line in the garage. Go from the garage through the elevator and into the baggage claim area. From the baggage claim area go through the escalator and into the lobby. Continue the line from the lobby through the moving sidewalk and into the waiting area. Pick yet another color and draw a line following the same route, but at the lobby go through the walking area to the waiting area.

6. Continue drawing lines to indicate the routes from the garage to the waiting area making sure to change colors for every line. The different colors will help the students to see if they have missed a possible route and will make it easier to count the lines when finished. If the students continue drawing their lines in an organized manner, completing the diagram will be much easier. There are 18 possible ways to get from the garage to the waiting area in this problem.

7. Other possible pictures and diagrams that might be used in problem solving are: Venn diagrams, maps, blueprints, and grids.

Evaluation The students should be able to construct their own appropriate picture and/or diagram needed to solve the problem and to use it to solve the problem.

Reteach Supply the students with the pictures or diagrams needed. Guide them through problems that contain fewer possible paths or routes to an area.

Extension Have students write equations for solving the problem. The airport problem contains 3 ways to go from the garage to the baggage claim area, 3 ways form baggage claim to the lobby, and 2 ways from the lobby to the waiting area. Therefore, $3 \times 3 \times 2 = 18$. The problem may also be extended by changing the numbers of possible ways from one area to another.

EE: ID *Related EE: IB* *TAAS Objective: 12A*

Objective The learner will use charts and graphs as appropriate strategies in problem solving.

Activity Create a chart that will facilitate solving a problem.

Materials Paper, ruler, pencil, tickets

Resources *The Problem Solver*, Creative Publications

Procedure

1. Guide students through the following problem:

   Lori has 30 carnival ride tickets. Riding the Ferris Wheel is 2 tickets, the Roller Coaster is 6, and the Zipper is 8 tickets. How many different combinations of rides can she enjoy?

2. Construct a chart containing 3 columns, one for each ride.

Zipper	Roller Coaster	Ferris Wheel

3. Select the ride that requires the most tickets. How many times can Lori ride the Zipper? Separate the amount of tickets required and record the number of rides possible on the chart. Are there any remaining tickets? What other rides could she ride with the remaining tickets? Record the results on the chart. HINT: Always record the greatest number possible for each ride before moving to the next combination.

Zipper	Roller Coaster	Ferris Wheel
3	1	0
3	0	3

4. Continue with the Zipper, recording one number less than the first two rows. Separate the tickets required. Are there any tickets left? What other rides could she ride with the remaining tickets? Record these on the chart. Remember to go in descending order.

Zipper	Roller Coaster	Ferris Wheel
3	1	0
3	0	3
2	2	1
2	1	4
2	0	7

5. Continue the same pattern to finish the list, separating tickets as needed. How many combinations of rides can Lori enjoy?

Zipper	Roller Coaster	Ferris Wheel
3	1	0
3	0	3
2	2	1
2	1	4
2	0	7
1	3	2
1	2	5
1	1	8
1	0	11
0	5	0
0	4	3
0	3	6
0	2	9
0	1	12
0	0	15

Evaluation Student performance on comparable problem.

Reteach Create a problem using only two columns and fewer combinations.

Ed has $1.00 to spend on some candy. He can purchase a candy bar for 50¢ or a lollipop for 25¢. How many combinations of candies can Ed buy?

50¢	25¢
2	0
1	2
0	4

Extension Use a variation of the listing solution.

Josh and Jaime are going to play a game to see who has to wash the dishes. Josh hands Jaime a die whose six faces are marked 1 to 6. To play, they take turns rolling the die 3 times in a row. The first one to roll a total of 12 in 3 rolls gets to dry the dishes instead of washing.
How many different ways can the boys combine 3 rolls of the die to total 12?

1. Set up a chart with three columns labeled Roll 1, Roll 2, and Roll 3. Begin with 1 for Roll 1; for Roll 2, record 5; and for Roll 3, record 6. Is there another way to arrange these numbers, using 1 for Roll 1?
2. Continue using 2 for Roll 1 and recording all possible combinations. There are six sides on the die so continue with 3 through 6 for Roll 1.

Roll 1	Roll 2	Roll 3
1	5	6
1	6	5
2	5	5
2	4	6
2	6	4
3	6	4

EE: 1D *TAAS Objective: 12A*

Objective The learner will take a given word problem and make it simpler to arrive at a solution.

Activity Solve a given word problem by making it easier.

Materials Sentence strip with Distance = Rate x Time

Procedure

1. Give each student the following problem: (on the overhead or handout)

   A car averages a distance of 180 miles in 5 hours. How fast is the car going?

   Introduce the distance formula to the students: Distance = Rate x Time

2. Have the students place the given information in the appropriate place on their strips. This can be done with a pencil.
   Distance = Rate x Time
   180 miles = ? x 5 hours

3. Tell students that the rate is missing. Ask if they know how the rate is found. Tell students to study the way the problem looks: 180 = ? x 5
   This is a multiplication problem. One of the factors is missing.

4. Give students this problem: 10 = ? x 5. Students will immediately know that the answer to this problem is 2. Ask students how the 2 came about. Ten was divided by five. Have students apply that simple knowledge to working 180 = ? x 5 In dividing, some students will not know what number is the divisor or the dividend. Have them go back to the simple problem of 10 = ? x 5. Which number was the divisor (the number outside the division sign)? The answer is 5, therefore:

   $5\overline{)180}$ just as: $5\overline{)10}$

   The solution is 36 miles an hour.

5. Give the students the following problem:

   A person travelled 195 miles travelling at a speed of 65 miles an hour. How long did it take the person to travel that far?

   Have students place given information under the appropriate place in the distance formula give. Distance = Rate x Time
   195 miles = 65 miles an hour x ?
   Students should note that a factor is missing again. 195 = 65 x ? Apply the simple knowledge of 10 = 2 x ? Have students work this problem. Students should divide 195 by 65 and get 3 hours.

Evaluation Give students five word problems where they will be using the distance formula. Three correct problems will indicate mastery, but check to see why the others were missed and reteach if necessary.

EE: ID *Related EE: ID* *TAAS Objective: 12A*

Objective The learner will work backwards as an appropriate strategy in solving problems.

Activity Using a circle graph or diagram, the students will work backwards in order to solve a problem.

Materials Paper, crayons, pencil

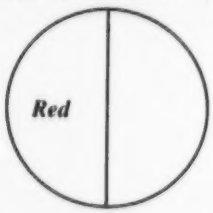
Resources *The Problem Solver*, Creative Publications

Procedure

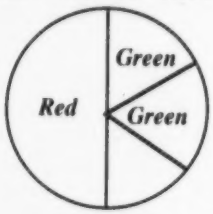
1. Assist the students in making a circle graph to solve the following problem:

   A crowd of people is watching the brave men wash the windows of the skyscraper. First the man in the red shirt washed one half of the windows in the building. Then the man in the green shirt washed two thirds of the windows that were left. Next the man in the blue shirt washed one half of the remaining windows. To everyone's amazement the little man in the yellow shirt washed the last 10 windows. How many windows did the brave men wash altogether?

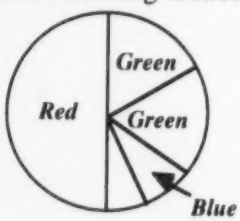
2. Draw a circle and divide it in half as stated by the number of windows washed by the man in red. Label this area by coloring it red.



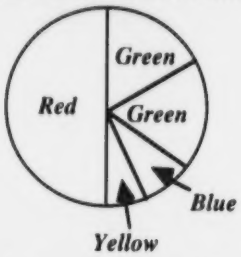
3. Divide the other half of the circle into thirds as stated by the man in the green shirt. Label two of the one third sections for the man in green by coloring it green.

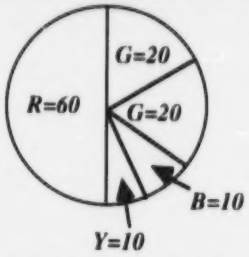


4. Divide the remaining section in half for the man in blue. Label this section with blue.



5. The last empty section is the 10 windows washed by the man in yellow. The students can then see that the section for the man in blue is the same size as the man in yellow, making the number of windows he washed also 10. The man in green washed two sections the size the man in yellow and the man in blue washed together. This takes up half of the circle, so the remaining half is the number of windows the man in red washed. The man in red washed the same number of windows that the other three men washed altogether.





With this process the students are using the information used at the end of the problem and working backwards to the beginning of the problem in order to solve the problem.

Evaluation Students should be able to determine the graph or diagram needed to solve the problem and solve the problems as needed.

Reteach Provide the students with the circle graphs or diagrams needed. Depending on the ability level, have the circle graphs or diagrams divided into the appropriate sections, or partially divided. Then assist the students in labeling the sections and working backwards to solve the problem.

Extension Increase the number of windows washed by the man in yellow. Provide a problem with a greater number of items.

EE: ID *TAAS Objective: 12A*

Objective The learner will use logic as an appropriate strategy in solving problems.

Activity Construct and utilize a matrix to solve logic problems.

Materials Paper, ruler, pencil

Resources *Logic, Anyone?* Fearson Teacher Aids

Procedure

1. Provide students with the following logic problem:

   Five students each enjoy a specific sport. No two children like the same sport. The children are Leo, Maria, Okari, Jessie, and Sam. Their favorite sports are soccer, hockey, baseball, football, and kickball. Which sport does each enjoy?

   1. Leo doesn't like soccer.
   2. Maria and Leo have never played football.
   3. Leo doesn't care for baseball.
   4. Okari has never played kickball.
   5. Sam is a great hitter and is able to practice his hitting a lot in his favorite sport.
   6. Leo and Sam do not like kicking balls because it hurts their toes.
   7. Okari likes soccer the best.

2. Guide the students in constructing a matrix that is appropriate for the problem. Because there are five students and five sports, the matrix will need to have five columns and five rows.

	soccer	baseball	hockey	football	kickball
Leo					
Maria					
Okari					
Jessie					
Sam					

3. Start with Rule 1. According to this rule, Leo doesn't like soccer. Place an X in the box across from Leo and under soccer to show that he will not pick this sport.

	soccer	baseball	hockey	football	kickball
Leo	X				
Maria					
Okari					
Jessie					
Sam					

4. Rule 2 states that Leo and Maria have never played football. It is logical that they will not pick football. Place an X in the box across from Maria and under football. Do the same for Leo.

	soccer	baseball	hockey	football	kickball
Leo	X			X	
Maria				X	
Okari					
Jessie					
Sam					

5. Rule 3 states that Leo doesn't like baseball. Place an X in the box across from Leo and under baseball.

	soccer	baseball	hockey	football	kickball
Leo	X	X		X	
Maria				X	
Okari					
Jessie					
Sam					

6. Rule 4 tells us logically that Okari will not pick kickball as her favorite sport. Place an X in the box across from Okari and under kickball.

	soccer	baseball	hockey	football	kickball
Leo	X	X		X	
Maria				X	
Okari					X
Jessie					
Sam					

7. Rule 5 tells us that Sam's favorite sport deals with hitting something. One can deduct his favorite sport is baseball. Write the word *yes* in the box across from Sam and under baseball. We now know Sam's favorite sport is baseball, place an X in every other box across from Sam's name. If Sam's favorite sport is baseball, we know that everyone else has a different favorite sport. Place an X in every empty box under baseball.

	soccer	baseball	hockey	football	kickball
Leo	X	X		X	
Maria		X		X	
Okari		X			X
Jessie		X			
Sam	X	yes	X	X	X

8. Rule 6 give us a lot of information. It tells us that Leo and Sam do not like sports that deal with kicking balls. Place an X in every box, across from their names, and under any sport that one must kick a ball while playing.

	soccer	baseball	hockey	football	kickball
Leo	X	X		X	X
Maria		X		X	
Okari		X			X
Jessie		X			
Sam	X	yes	X	X	X

9. One can now see that Leo's favorite sport is hockey. Write *yes* in the empty box across from Leo's name. Place an X in every empty box under hockey because we know that no one else will pick hockey.

	soccer	baseball	hockey	football	kickball
Leo	X	X	yes	X	X
Maria		X	X	X	
Okari		X	X		X
Jessie		X	X		
Sam	X	yes	X	X	X

10. The last rule will give us the rest of the information needed. It tells us that Okari's favorite sport is soccer. We can now write *yes* in the box across from Okari and under soccer. Place an X in the empty box across from Okari's name. The empty boxes under soccer can also be filled in with an X. This leaves Maria with kickball, write *yes* in this box. Place an X in any empty boxes under kickball. The only empty box left tells us that Jessie likes football. Write *yes* in this box.

	soccer	baseball	hockey	football	kickball
Leo	X	X	yes	X	X
Maria	X	X	X	X	yes
Okari	yes	X	X	X	X
Jessie	X	X	X	yes	X
Sam	X	yes	X	X	X

11. Logic problems may also be solved by using Venn diagrams and syllogisms.

Evaluation Have the students construct and complete a matrix to help them solve logic problems.

Reteach Provide the students with a matrix that has been partially filled in after the number of items in the problem and the clues or rules are given.

Extension More detailed logic problems, containing clues with more deductive reasoning are more challenging. Students enjoy the challenge of proving syllogisms valid or invalid.

- All dogs are barking animals.
- All poodles are dogs.
- Therefore, all poodles are barking animals.
  (Valid)

- All books are big.
- All trees are big.
- Therefore, all books are trees.
  (Invalid. It's not stated big things are only books.)

EE: 2C *TAAS Objective: 2E*

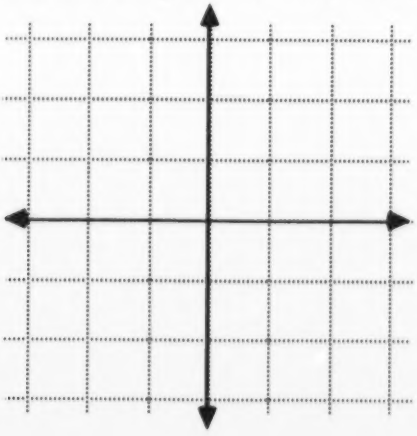
Objective The learner will build simple functions using concrete models and will generate a corresponding rule (graphics and ordered pairs).

Activity Construct a simple grid with **x** and **y** axis on the floor of the classroom and be able to find given points.

Materials Masking tape, construction paper circles 3 inches in diameter (have various ordered pairs)

Procedure

1. Have the students assist you in making a grid using masking tape on the floor of the classroom or other appropriate location. Direct the students in the making of the grid. Use your discretion as to the grid size. If the floor has tile, use the tile as a guide.



2. Explain to the students the different parts of the grid. The horizontal line going from left to right is called X. The vertical line going up and down is called Y. Ordered pairs are two numbers within a parenthesis separated by a comma that indicate a point on the grid. Tell students that to find a point on the grid, always start at zero.

3. Have a student find the point (3,4) on the grid. Assist this first student in the location of the point. Have the student go to zero and stand there. He or she then proceeds to the right 3 spaces. From that location he or she goes up 4 spaces. This final point is (3,4). Have the student place the red circle with the ordered pairs (3,4) written on it at that point.

4. Have another student proceed to locate point (3,2) on the grid. Make sure the student goes to zero first. From zero he or she moves 3 spaces to the right and up 2. Have this student place the circle with ordered pairs (3,2) written on it at that point. Students who are not moving along the grid may all in chorus direct the student on the grid where to move. For example, have a student stand at zero. Show the class a construction paper circle having the ordered pairs (2,4) written on it. The whole class would then instruct the student on the grid by saying together, "To the right, one two, stop. Up, one, two, three, four, stop." Music or rhythmic verses can be used to further motivate students.

Evaluation For evaluative purposes, have the students find given ordered pairs on a paper grid.

Reteach Pair students who have mastered the activity with students who have not. Peer-tutoring or one-on-one instruction may be utilized for reteaching.

Extension Students may be introduced to negative numbers on the grid. The floor grid may be used. Have students locate points like: (-3,3). The student stands at zero and goes to the left 3 spaces and then 3 up. For (-2,-3), the student would start at zero and go 2 spaces to the left and 3 spaces down.

EE: 2C *Related EEs: 3D, 4G, 7C* *TAAS Objective: 2C*

Objective The learner will build simple functions using concrete models and generate a corresponding rule to develop a concept of ratio and application problems.

Activity Build models of ratio problems and use them to develop the idea of equal ratios.

Materials Paper, pencil, red and blue connecting cubes, various other items (crayons, pens, paper clips, etc.)

Procedure

1. Have the students use connecting cubes to solve proportions and ratios. Show the students 3 blue cubes and 4 red cubes. Tell them that for every 3 blue cubes, there are 4 red cubes. Ask: If you added another group of 3 blue cubes, how many red cubes would you need to add to have 4 red cubes for every 3 blue cubes? How many red cubes would you then have? (8)

$$\frac{3 \text{ blue cubes}}{4 \text{ red cubes}} = \frac{6 \text{ blue cubes}}{8 \text{ red cubes}}$$

   Making 2 groups of 4 is the same as multiplying both numbers by 2.

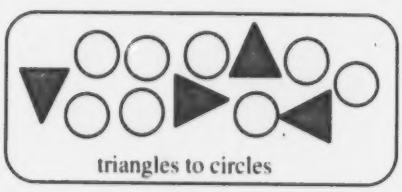
2. Change the number of red and blue cubes for other problems. Make ratios of other items such as: crayons to pens, paper clips to glue bottles, and books to pencils.

Evaluation Have the students complete a teacher-made worksheet dealing with naming the ratio of items and solving equal ratios.

Reteach Students work in pairs to complete the following activity:

Materials: 20 index cards

Write these ratios on 10 cards: 1/2, 2/3, 3/1, 1/4, 4/3, 2/5, 5/4, 6/1, 5/6, 3/8. On the other 10 cards, draw groups of geometric shapes to illustrate the ratios and write the ratios to be named. Students match pairs of picture cards and ratio cards.



triangles to circles

Extension Have students design a blueprint of their ideal home. Using a key to state the ratio of actual size to the measurements of their blueprint will show application of ratio and proportion.

EE: 3AB *Related EE: 3C* *TAAS Objective: 1C*

Objective Using pie and bar graphs, the learner will illustrate given percents.

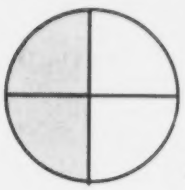
Activity Using pie and bar graphs, the learner will illustrate given percents.

Materials Chalkboard, colored chalk (if available)

Procedure

1. Explain to the students that the symbol % means percent. Tell students that 50% means 50 per hundredths or 50/100. If 50/100 is reduced to lowest terms, it is 1/2. Therefore, 50% = 50/100 = 1/2. 50% of something is 1/2 of it.

2. Illustrate for the students:



The shaded part is 1/2 of the whole pie or 50% of it.

3. Explain to the students that 25% means 25/100. If 25/100 is changed to lowest terms, it is 1/4. Therefore, 25% = 25/100 = 1/4. 25% of something is 1/4 of it.

4. Illustrate for the students:

The shaded part is 1/4 of the whole pie or 25% of it.

5. Explain to the students that 75% means 75/100. If 75/100 is changed to lowest terms, it is 3/4. Therefore, 75% = 75/100 =3/4. 75% of something is 3/4 of it.

6. Illustrate for the students:

The shaded part is 3/4 of the whole pie or 75% of it.

7. Have students now draw bar graphs illustrating 50%, 25%, and 75%. Example: 50% = 50/100 = 1/2, therefore

= 50% shaded

Evaluation Use the bar graphs as an evaluative tool.

Reteach If students are having difficulty with this lesson, check to see that they are able to reduce. Otherwise, have students go over the lesson at a slower pace.

Extension Have the students illustrate percents using bar graphs of: 85%, 65%, and 55%. Sample: 85% = 85/100 = 17/20 =

shaded part = 85%

EE: 3B *TAAS Objectives: 1C, 2F*

Objective The learner will explore relationships between whole numbers and fractions.

Activity Work individually or in pairs to explore relationships between whole numbers and fractions.

Materials Index cards cut in half

Procedure

1. Tell students that all whole numbers can be expressed as fractions by placing a one under the whole number: 2/1. By finding the multiple of the numerator and denominator, you discover other fractional names for 2: 2/1 = 4/2, 6/3, 8/4, 10/5, 12/6, 14/7, ... Have students find fractional names for 5 following the steps mentioned above: Change the 5 to a fraction by placing a one under it. Find multiples of 5 and 1 and write other fractional names for 5: 5 = 5/1 = 10/2, 15/3, 20/4, 25/5, 30/6, ...

   Alert students to the fact that in all the fractions written above when the numerator is divided by the denominator the answer will be the whole number. In other words, 10/5 means 10 divided by 5 which is 2.

2. Give each student a whole number. Ask each student to find at least five fractional names for that whole number. Have students write the fractions on the index card halves provided. Pick up the cards when the students have finished. Mix the cards up and give one to each student. Have each student tell which whole number is represented by the fraction given him or her. Remind students to divide the numerator by the denominator to get their whole number.

Evaluation Use the activity with the cards to evaluate whether students have understood lesson.

Reteach Should students have difficulty with this lesson, make sure that they understand multiples. To find the multiples of 5, for example, they are to count by 5's. The multiples of 5 are 5, 10, 15, 20, etc. The multiples of 1 are 1, 2, 3, 4, etc.

Extension Have students find fractional names for two- and three-digit numbers. Have students place these on index card halves. Mix these up and give each student five cards face down. Have students compete to see who can come up with the five whole numbers represented by the fractions. You may want to make sure that each student gets an equal number of two- and three-digit whole numbers represented.

EE: 3E *TAAS Objective: 1A*

Objective The learner will compare and order positive rational numbers.

Activity Compare and order decimals.

Materials Index cards with a decimal written on each

Procedure

1. Write the following decimals on the chalkboard or overhead:

   .5 .50 .345 .99

2. Ask students to guess which is largest. Most students will say that .345 is largest. By annexing zeros, one can make all the decimals thousandths. By comparing the numbers now, one can see that .99 is largest:

   .500 .500 .345 .990

3. Write the following decimals on the board or overhead. Ask students to guess which is smallest:

   .1 .23 .001 .1201 .26

4. Most students will be tempted to say that .1 is the smallest. Have students annex zeros to make the decimals all in the ten-thousandths place:

   .1000 .2300 .0010 .1201 .2600

   Now one can see that .001 is the smallest.

Evaluation Give each student an index card with a decimal written on it. Divide the students into groups of fours. Have each group arrange its index cards with decimals from least to greatest. When all groups are finished, have each group share results with the whole class.

Reteach Use peer tutoring for reteaching.

Extension Have the whole class arrange their index cards with decimals from least to greatest. One rule: NO TALKING.

Objective The learner will develop the meaning of integers as representations in problem situations.

Activity Students will learn that integers can be either positive or negative. Students will open an imaginary bank account within the classroom.

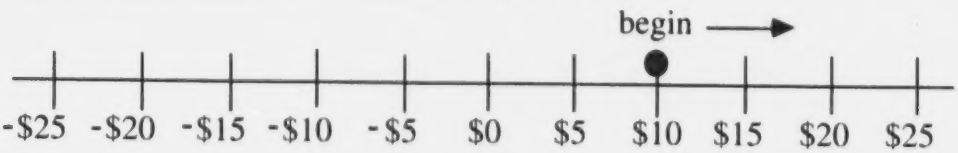
Materials Play money, checks (adapted), receipt books

Procedure

1. Tell students that at the end of each week, each student will be given a pay check. Students will be paid $4.25 an hour for a total of seven hours each day. Income tax @ 15%, and Social Security @ 7% will be deducted from the paycheck. Fines given during the week or days absent will also be deducted.

2. Explain to students the difference between gross pay and net pay. Gross pay is a positive amount. Income tax, Social Security tax, fines, and the amount taken for days absent are negative amounts which are subtracted from the gross amount. The result is the net pay. Deposits are positive amounts that are added to balances.

3. Draw a figure as indicated with increments of fives:



4. Follow through with students the following scenario using this illustration.

   John has $10. (Point to the $10.) John received $5.00. (Go up 1 to $15.00.) John spends $15. (Go down 3.) John deposited $20.00. (Go up 4.) John made a check for $25.00. (Go down 5.) How much money does John now have? He owes $5.00. He's at -5.

5. Allow students to live in an imaginary city. Every Friday or once a month, students receive a paycheck that they can deposit in the bank or cash for play money which in turn they can use to buy articles from each other. Students may create their own companies. Have students represent a bank, light company, gas company, landlords, etc. Have students experience the making of a budget.

Evaluation Student turns in a report on how money was spent indicating gains and losses.

Reteach Work with each student individually and assist him or her in making a budget or putting money to work. Encourage students to put up a small store or open a loan company that charges interest for loans. Be creative.

Extension Students may use the newspaper to buy stocks and keep track of losses and gains.

EE: 4B *Related EE: 4A* *TAAS Objective: 12A*

Objective The learner will use the order of operations to solve multi-step problems, using a calculator when appropriate.

Activity Solve a variety of multi-step problems using the order of operations.

Materials Teacher-made handouts with a variety of multi-step problems using the order of operations.

Procedure

1. Write the following multi-step problem on the chalkboard for the students to see:

   $4 + 5 - 32 \times 5 \div (6 - 1) =$

2. Model for the students how to solve this problem correctly. Stress the importance of following the order given below:
   1. Work any powers.
   2. Solve what is inside the parentheses.
   3. Multiply or divide (whatever comes first from left to right).
   4. Add or subtract (whatever comes first from left to right).

3. Follow each step as you work the problem on the board for the students:

The problem is	$4 + 5 - 3^2 \times 5 \div (6 - 1) =$
Work the powers	$4 + 5 - 9 \times 5 \div (6 - 1) =$
Solve what's inside the parenthesis	$4 + 5 - 9 \times 5 \div 5 =$
Multiply or divide (whatever comes first)	$4 + 5 - 45 \div 5 =$
Multiply or divide (whatever comes first)	$4 + 5 - 9 =$
Add or subtract (whatever comes first)	$9 - 9 =$
	0

4. Give students the following problem to solve. Have them follow each step in solving the problem:

$$4 = (3 \times 5) + (5 \times 1) + 32 - 3 =$$

Note: After students solve the powers, they solve the problems inside the parentheses. First one, then the other.

Evaluation Give students a handout with several multi-step problems to solve. Use this to evaluate how students are understanding the lesson.

Reteach Given a multi-step problem, have the student follow each step carefully. Step one states to work any powers. Have the student work any powers. Note that there may be more than one number with powers to be worked out. The same holds true for each of the other steps. Stress this to all students but more so to students having difficulty.

Extension Tell students that some calculators are programmed to work multi-step problems automatically. In other words, students need only input numbers and operations as they appear. Other calculators are not programmed to do multi-step problems. With these, the order of operations becomes important. For enrichment, expose students to both kinds of calculators. Have students construct problems that they can use to test the calculators. A calculator that can do multi-step problems correctly is called a calculator with algebraic logic.

EE: 4G *Related EEs: 3D, 4H, 7C* *TAAS Objective: 2C*

Objective The learner will estimate and solve problems using ratios and proportions.

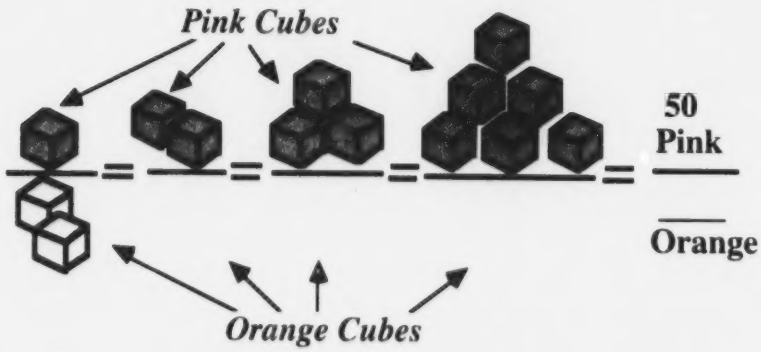
Activity Utilizing cubes, set up proportions for number ratios.

Materials Paper, store advertisements, pencil, rainbow cubes, road maps

Resources *Algebra Thinking: First Experiences*, Creative Publications

Procedure

1. Guide the students through the process of finding equal proportions by using rainbow cubes. Provide pairs of students with cubes and various problems similar to the example:



2. By arranging the cubes like those given and filling in the cubes needed to complete the ratios, the students are able to visualize what the ratio and its equal proportion looks like.

3. Once the students understand the concept of using the cubes to find the proportions, they are able to understand the process needed. Have the students use store advertisements to write ratios for the products and their prices. These ratios can then be used to determine the cost of buying fewer items or a greater number of items.

   Example: A package of notebook paper containing 100 sheets cost 78¢. What would the cost be for a package containing 250 sheets?

   $$\frac{100}{78¢} = \frac{250}{?}$$

   (Remind the students that the cross products are equal.)

4. Road maps may also be utilized to show ratios and proportions. Use a Texas road map or one from any other state that might interest the students. Have the students write a ratio using the mileage scale and write the actual mileage from one city to the next by measuring the distance on the map.

Evaluation Students will write ratios for given problems and correctly restate the ratios as proportions on a teacher-made worksheet using their manipulatives.

Reteach Provide the students with worksheets that contain parts of the proportion; step by step, walk through instructions.

$$\frac{5}{6} = \frac{?}{12}$$

$5 \times 12 = ? \times 6$

$60 = ? \times 6$

$60 \div 6 = ?$

$$\frac{6}{22} = \frac{15}{?}$$

$6 \times ? = 15 \times 22$

$6 \times ? =$ _____

$? =$ _____ $\div 6$

Extension Provide the students with a worksheet on sharing profits. Explain that when individuals join together to start a business, they need to agree on how to share the profits.

Lori and Marty agree to share the profits from their ice cream shop in the ratio of 3 to 2.
How do they share a $600 profit?
They agreed upon a 3 + 2, or 5 equal shares.
Lori will receive 3 of the 5 shares.
The ratios of Lori's share to the total number of shares is 3 to 5.

$\frac{3}{5} = \frac{?}{600}$ Lori receives $360.

Marty's share is 2 to 5.

$\frac{2}{5} = \frac{?}{600}$ Marty receives $240.

Once the students are comfortable finding shared profits, have them write problems of their own.

EE: 4H *Related EEs: 7E, 7F* *TAAS Objectives: 2C, 11D*

Objective The learner will use ratios to solve probability problems.

Activity Use a variety of manipulatives to solve probability problems.

Materials Paper, pencil, bags containing manipulatives (colored cubes, marbles, candies)

Resources TEA Module 10, *6-8, Probability, Statistics, and Graphing*

Procedure

1. Prepare a box filled with colored candies. The oval-shaped colored hard candies work well as they will show up in their color if placed on the overhead.

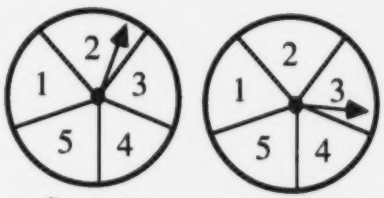
2. Place an assortment of the candies on the overhead so the students are able to see how many of each color is being placed in the box. Put the candies in the box so that the students are not able to see them.

3. Starting with one student, have students predict what color candy they will draw from the box. Guide them to see that the color containing the most pieces will probably be drawn. Discuss and write a ratio to show the probability of the student choosing that color candy. Stress that the numerator will be the number of that color of candy and the denominator will be the total number of candies.

4. The student draws a piece of candy to see if his or her prediction is accurate. Continue until every student has drawn a piece of candy. The students will see that the ratio written to show the probability changes with each piece of candy drawn. You may also want to have the students reduce the ratios.

5. The students should now be familiar with the process needed to write ratios to show probabilities. Give each pair of students, or group of three, a paper bag containing items such as: colored cubes, marbles, small construction paper shapes, or magazine pictures of clothing. The students will write ratios to predict the probability of drawing certain items from their paper bags. After each item has been drawn, they may replace it in the bag before drawing again or set it off to the side. Guide the students through some examples.

Evaluation Provide students with a worksheet containing pictures of two spinners.



first spin second spin

Ask students to find the probability of five events if each spinner is spun one time.

Example:
What is the probability of a product that is a 6?
The students will need to find all the possible ways to make a product of 6 using the two spinners. They will find that they can spin a 2 with the first spinner and a 3 with the second or a 3 with the first and 2 with the second, making 2 chances out of 25 that they will spin a product of 6. The worksheet may contain as many problems as deemed necessary.

Reteach Provide the students with spinners and paper bags containing rainbow cubes. Have the students predict the probable number of favorable outcomes for each of the following:

- drawing your favorite color
- spinning your favorite number
- spinning your favorite number and drawing your favorite color

Extension Find probabilities using a calendar.

Provide the students with a calender of any given month.
Ask questions such as:

- If a date is selected at random, what is the probability that it will be divisible by 5?
- If a date is selected at random, what is the probability that it will be a multiple of 8?
- Frank Garcia's art class meets on Wednesdays and Fridays. He only attends if the class meets on a prime number date. What is the probability that Frank will attend an art class during the month?
- What is the probability that a date that is a square number will fall on Sunday? (The day of the week may need to be changed to correspond with the month being used.)

Objective The learner will write and solve simple linear equations from problem situations and check the reasonableness of the result.

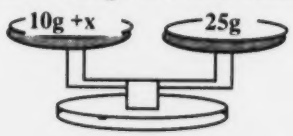
Activity Use equations to solve problems.

Materials Balance beam (optional)

Procedure

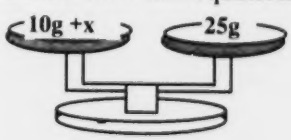
1. Give the students the following situation:
   Object A weighs 10 grams.
   Object B weighs _____.
   Together Object A and B weigh 25 grams.
   How much does Object B weigh?

2. In using the balance beam, place 25 grams on one end of the scale and 10 grams on the other side. Add grams until balance is achieved. Note how much was added.

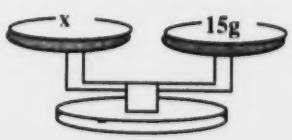


3. Have students watch you model the following on the chalkboard or overhead:
   Object A + Object B = 25 grams
   10 g + x = 25 grams

4. Have students view this equation as a balance scale:



Ask the students what will happen if 10 grams are subtracted from both sides of the scale, then:

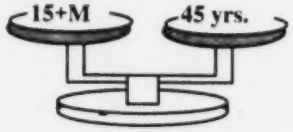


The students should discover that the reason you subtract 10 from both sides is to keep the scale balanced.

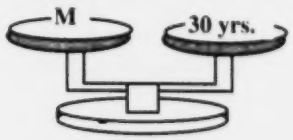
5. Follow up with the problem. Have the whole class assist in solving it using the drawn scale:

   Sally is 15 years old. Sally and Mary together are 45 years old. How old is Mary?

   Sally's age + Mary's age = 45 years



   Ask the students what they can do to solve the problem.. The response should be "Subtract 15 from both sides."



Evaluation Have students work more problems similar to the above. Monitor students for evaluative purposes.

Reteach Deal with students having difficulty on a one-to-one basis.

Extension Addition is the opposite of subtraction.
Subtraction is the opposite of addition.
Have students work problems such as:

m + 18 = 20, m - 20 = 60 by doing the opposite:

$$\begin{array}{r} m + 18 = 20 \\ \underline{-18 \quad -18} \\ m = 2 \end{array} \qquad \begin{array}{r} m - 20 = 60 \\ \underline{+20 \quad +20} \\ m = 80 \end{array}$$

EEs: 5B, 5C, 5D *TAAS Objective: 4E*

Objective The learner will use manipulatives and apply formulas to solve problems involving area.

Activity Find the area of irregular figures using geoboards.

Materials Paper, geoboards, pencil, cookies

Resources HBJ *Problem Solving*; TEA Math Module 16, *6-8, Geometry*

Procedure

1. Provide the students with illustrations of various irregular shapes. Have the students duplicate the shapes on their geoboards and find the area of each shape.

2. Have the students work in pairs to see if they can find another way to solve the area problems that doesn't involve counting the square units.

3. Provide the students with illustrations of common polygons. Have the students duplicate the shapes on their geoboards and find the area of each shape.

4. Have the students work in pairs to see if they can find another way to solve the area problems. Introduce the formulas for area.

5. Introduce the formula for area of a circle. Have the students practice finding the area of circles by finding the area of something that interests them. Cookies or some other round treat works well. They must find the area before they eat the item.

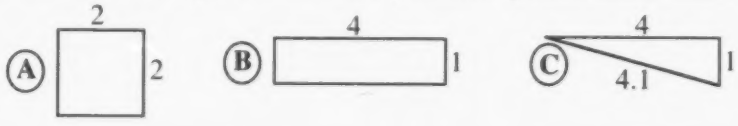
Evaluation Students will solve application problems involving area. Provide a teacher-made worksheet containing application problems.

Example: Joe has a square piece of lawn 30 feet wide.
The area of his lawn is _____.

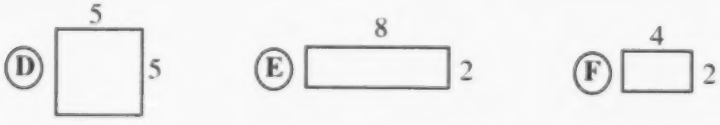
Reteach Provide the students with illustrations of various shapes. The students will fill the insides of the illustrations using rainbow cubes.

Extension Students will use clues to find the correct geometric figure.

- It has an area of 4 square units and a perimeter greater than 9 units.



- It has a perimeter that is not a perfect square, an area that is a perfect square, and the number of square units of area is greater than the number of units in the perimeter.



Objective The learner will be able to use models to develop and apply the formula for the volume of rectangular solids.

Activity Construct rectangular models using linking blocks. Students will then find the volume of these blocks. Students will also determine the volume of various objects found in the classroom.

Procedure

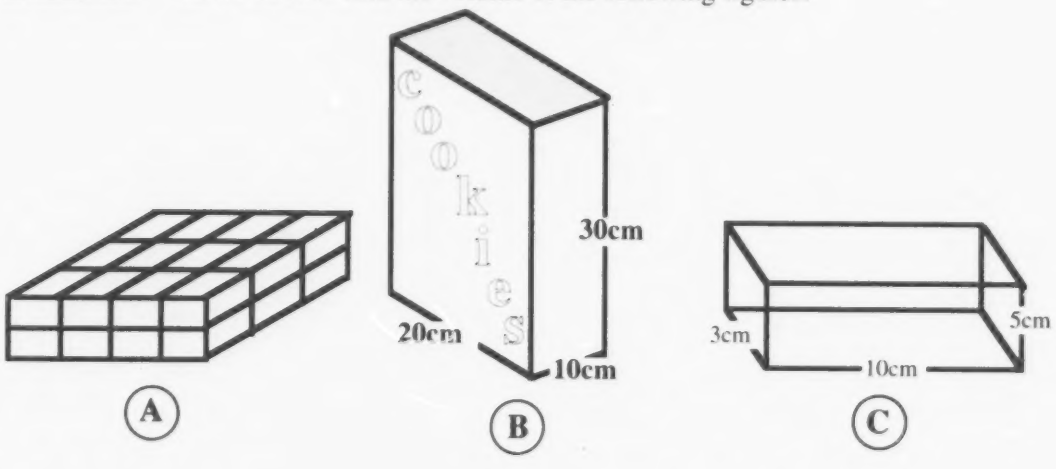
1. Explain to students that volume is the space inside something. For example, the volume of the classroom is the space inside the whole room.

2. Have students compare the volume of the classroom to the volume of the gym or cafeteria. Students can see that the volume, or space, of the classroom is smaller than that of the cafeteria or gym. Explain to students that the way volume is measured is in cubic units. If we are measuring a room's volume, in cubic feet what we are trying to find out is how many cubes which are one foot in length, one foot in width, and one foot in height will fit inside the room. To find this out, multiply the length of the room times the width of the room times the height of the room. If the room is 30 feet by 20 feet by 8 feet, then the volume of the room would be 30 feet x 20 feet x 8 feet = 4800 cubic feet.

3. Have the students use linking blocks that are 1 inch by 1 inch by 1 inch to construct a rectangular solid. After the solid is completed, ask students to multiply the number of blocks in the length times the number of blocks in the width times the number of blocks in the height. Have students compare the number of blocks in the whole rectangular solid and the answer they got when multiplying the length by the width by the height. The computed answers should be the same as the results when they counted the cubes.

4. Have students measure different boxes on display in the room. Students are to find the length, width, and height of each box in inches or feet. Measurements should be to the nearest foot or inch. After measurements are completed, have the students find the volume for each box. Stress that students are to indicate cubic inches or cubic feet after each answer. This shows that volume is indicated.

Evaluation Have students find the volume of the following figures:



Objective The learner will measure and compare angles.

Activity Measure and compare angles.

Materials Protractors, handout, straightedges (rulers), paper, scissors

Procedure

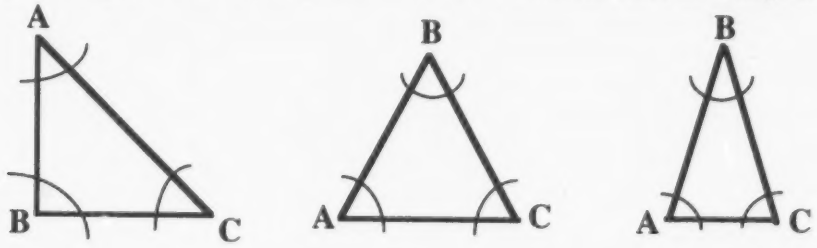
1. Demonstrate to students using the overhead projector how to measure angles using the protractor.

   To measure an angle, place the center mark of the protractor on the vertex of the angle and the zero mark on a side of the angle. You may start by measuring a right angle. Right angles are 90 degrees.

2. Have students look around the room and find 90-degree angles. The corners of their paper will have 90 degrees. The corner of the doors, windows, tiles, etc. will have 90 degrees. Measure angles which are smaller than 90 degrees.

3. Tell students that angles that are less than 90 degrees are called acute angles. Have students find acute angles in the room. These may be difficult to find. Have students make a peace sign using index and middle finger. The result is an acute angle.

4. Tell students that angles that measure more than 90 degrees are called obtuse angles. Have students find obtuse angles in the room. Allow students some time to measure several given angles. Give students a handout with different sizes of triangles.



5. Have students measure the angles of the triangles. Have students add up the 3 measurements for each triangle. Students will be surprised to discover their answers will be close to 180 degrees.

6. Let students cut out several triangles, tear off the angles and place them together to form 180 degrees.

Evaluation Assessment is on-going while students measure angles. Monitor students closely.

Reteach Have students pair up and ask each to make different angles with the partner measuring each. Have each evaluate the other.

Extension Have students find the total measurement of the angles of a triangle, square, pentagon, hexagon, septagon, octagon, nonagon, decagon.

EE: 6B *TAAS Objective: 3B*

Objective The learner will visualize and sketch the results of a rotation, translation, or reflection using manipulatives.

Activity Draw and identify the rotation, translation, or reflection of an object.

Materials paper, small objects of various shapes, pencil, attribute blocks

Resources Addison-Wesley: *Mathematics* sixth grade text; HBJ *Manipulatives & Problem Solving*

Procedure

1. Provide the students with the following information:
   - When you slide the figure, the new figure is called a translation image.
   - When you flip the figure over a line, the new figure is called a reflection image.
   - When you turn the figure around a point, the new figure is called a rotation image.

2. Working in pairs, students are to find the rotation, translation, or reflection of attribute blocks. One student will trace the attribute block. He or she will then rotate, translate, or make a reflection of the block and trace it again. The second student will determine what type of image it is by rotating, translating, or reflecting the attribute block to its new position. Students will then change roles.

3. Once the attribute blocks have been used, have students follow the same procedure using other small objects (paper clips, little flags, erasers, etc.).

Evaluation Provide the students with a sheet containing several figures and their images. Have the students name the type of image it is.

Reteach Work with students individually or in small groups. Move attribute blocks or other objects to their new images and have the students imitate your movement. Discuss what type of movement was made and what the new image is.

Extension Using graph paper and a straightedge, have students draw a figure of their own creation. Then use translation, rotation, and reflection, alone or in combination, to change the position of the figure. The partner will try to figure out the combination used to change the position of the figure.

EE: 6C *Related EE: 6F* *TAAS Objective: 3A*

Objective The learner will build models of three-dimensional figures such as pyramids, cones, and prisms with polygonal bases and investigate the properties associated with these figures.

Activity The learner will construct three-dimensional figures and separate these into pyramids, cones, and prisms. Discussion on similarities will take place among learners.

Materials Scissors, paper, pencil, glue, two-dimensional patterns of three-dimensional figures on construction paper ready to be cut out.

Procedure

1. Have students get necessary supplies: scissors, paper, pencil, glue.

2. Give each student construction paper with figures to be cut out. Have the students cut out designs and construct each figure.

3. After students have completed their figures, have students separate figures into either: cones, pyramids, or prisms.

4. Conduct a classroom discussion on the properties of the cone, the pyramid, and the prism. Have students note the differences between each. Have students note particularly the differences between pyramids and prisms. Have the students note the similarities between each of the figures. In discussing the similarities and differences, have the students use terms such as faces, edges, and vertices. Explain to the students that by looking at the base of the figure they can tell the name of the figure. For example, if the prism has a square base, then the figure is called a square pyramid. Familiarize the students with the vocabulary by telling them that faces are the sides of the figure. A rectangular prism, for example, has six faces or sides. Edges of a figure are that part of the figure where two faces meet. A cube, for example, has 12 edges. The vertices are the corners found in the figures. A cube has eight vertices or corners. It is important that students know the vocabulary terms in describing similarities and differences.

Evaluation For evaluative purpose have the students complete the following chart:

Cones	Pyramids	Prisms
Characteristics	Characteristics	Characteristics
1.	1.	1.
2.	2.	2.
3.	3.	3.
4.	4.	4.
5.	5.	5.

Reteach Work individually with students who exhibit nonmastery. You may wish to elaborate on the characteristics of a prism in contrast to those of a pyramid. Stress that a pyramid is pointed; a prism is not.

Extension Have the students research the correct names of the three-dimensional figures constructed.

EE: 7E *Related EEs: 4H, 7F* *TAAS Objectives: 5A, 5B*

Objective The learner will determine and interpret fractional probabilities of simple events.

Activity List possible outcomes and give the probability of given events.

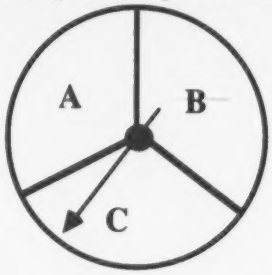
Materials Paper, coins, paper bag, rainbow cubes, pencil, number cubes (use cubes with a variety of sides), spinners

Resources *Mathematics in Action*: *Problem Solving Cards* , 1991, Macmillan/McGraw-Hill

Procedure

1. Provide each group of students with one of each type of manipulative to be used.

2. Students will list all the outcomes and give the probability of each outcome. They will then make predictions using their probabilities.
   Example: Write the probability of the spinner landing in region A. (1/3)



How many times would you expect to land in region C in 24 spins? (8)

3. Follow the same procedure for each manipulative used.

4. Count the number of times the letter "e" appears in a sentence in one of the textbooks. Predict how many times the letter "e" will appear on the page.

Evaluation Provide the students with the following information:

A paper bag contains 5 blue cubes, 3 red cubes, and 2 green cubes. Without looking, you reach into the bag and pick a cube. Record its color and replace it in the bag. In 20 draws, how many times would you expect to pick a blue cube? A red cube? A green cube? How many of each color if you were to pick a cube 100 times?

Reteach Provide the students with manipulatives and worksheet that contains part of the ratio and prediction given.

Extension Use an imaginary table containing weather information for the past 25 years. List the average number of days for each month that had sun, clouds, or rain. Ask various probability questions for the information listed on the table.

Example: Which month should we schedule a trip to the beach?
What is the probability of rain on any day of that month?

EE: 7F *Related EEs: 4H, 7E* *TAAS Objective: 5D*

Objective The learner will compare theoretical probabilities and experimental probabilities of an experiment.

Activity Make predictions based on probability and test their accuracy by experimenting.

Materials Paper, two-sided colored chips, pencils, colored chips (red, blue, yellow), number cubes, beans

Resources *Using Cooperative Learning to Enhance Your Math Instruction*
by Marny Sorgen, Bureau of Education and Research

Procedure

1. Provide each group of students with a number line numbered 2 to 12 and 11 beans. The students will place the beans on their number line, as many beans on a number as they would like, to predict the numbers that are rolled on two die.

2. Roll the two die and announce the sum. The students will remove one bean if they have one on the number announced. Continue until one group has removed all of its beans. The students will see that the sum with the most possibilities is 7. Play the game again and see if the groups place most of their beans on the middle numbers.

3. Provide each group of three with the following rules.

   Students in each group will play Paper, Scissors, Rock (without the slapping of wrists).
   - Player A gets 1 point if all show the same sign.
   - Player B gets 1 point if only two show the same sign.
   - Player C gets 1 point if all show a different sign.

4. Each player chooses a letter to be and begins playing, keeping track of points as he or she plays. Allow the groups to play until one player reaches 25 points. Discuss with the students whether this is a fair game. They will see that the probability of player B winning is the highest.

5. Allow them to play again after they have come up with a new point system to make the game fair.

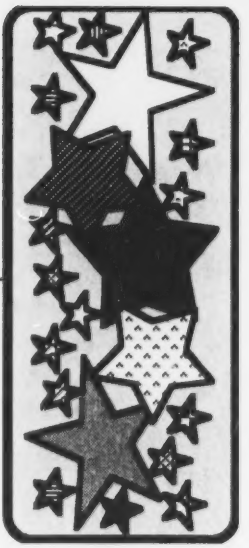
6. Draw a racetrack. Use colored cubes as race cars. The students will roll two number cubes to determine who will go first. The object is for the players to pick the race car that has the highest probability of moving.

   A player will roll the number cubes and toss both two-color counters.
   Using the rules, move the appropriate car. More than one car may move in a turn.
   - The blue car moves if the sum of the number cubes is 7 or more.
   - The red car moves if the two-color counters land with the same color sides up, but not if doubles are rolled.
   - The yellow car moves if both numbers on the cubes are even or if the two-colored counters are both red.

   The player whose car is the first to make a lap wins.

Evaluation Teacher observations and completed results from the games.

Evaluation



Philosophy

NCTM's *Professional Standards for Teaching Mathematics* and *Curriculum and Evaluation Standards for School Mathematics* (the *Standards*) emphasize the connection between assessment of students and analysis of instruction. In other words, mathematics teachers should monitor students' learning (both formatively and summatively) in order to assess and adjust teaching. Teachers must observe and listen in order to tailor teaching strategies. Information about what students are understanding should be used to revise and adapt short- and long-range plans, and students' understandings should guide teachers in shaping the learning environment. Also, teachers are responsible for describing students' learning to administrators, parents, and students themselves.

Students' mathematical power depends on various understandings, skills, and dispositions. The development of students' abilities to reason mathematically—to conjecture, justify, and revise based on evidence and to analyze and solve problems must be assessed. Students' dispositions toward mathematics (confidence, interest, perseverance, etc.) are also a key dimension that teachers should monitor.

The importance of using assessment to improve instruction is crucial. Information should be gathered from multiple sources using numerous assessment techniques and modes that are aligned with the curriculum. Assessment techniques must reflect the diversity of instructional methods implied in the *Standards* and the various ways students learn and process information. Instructional decisions should be based on this convergence of information from different sources.

Types of Evaluation

While paper and pencil tests are one useful medium for judging aspects of students' mathematical knowledge, teachers need information gathered in a variety of ways and using a range of sources. Observing, interviewing, and closely watching and listening to students are all important means of assessment. While monitoring students, teachers can evaluate the learning environment, tasks, and discourse that have been taking place. Using a variety of strategies, teachers should assess students' capacities and inclinations to analyze situations, frame and solve problems, and make sense of concepts and procedures. Such information should be used to assess how students are doing, as well as how well the tasks, discourse, and environment are fostering students' mathematical power and then to adapt instruction in response.

Principles relevant for judging assessment instruments at all levels also apply to program evaluation. Inherent in the *Standards* is an assumption that all evaluation processes should use multiple assessment techniques aligned with the curriculum and consider the purpose of assessment. Mathematics education described in the *Standards* places new demands on instruction and forces a reassessment of the manner and method by which students' progress is charted. Testing instruments must reflect the scope and intent of the instructional program to have students solve problems, reason, and communicate, and they must enable teachers to understand students' perceptions of mathematical ideas and processes and their abilities to function in mathematical contexts. Also, the testing instruments must be sensitive enough to help teachers identify individual areas of difficulty in order to improve instruction.

Available assessment techniques suggested in the *Standards* include multiple-choice, short-answer, discussion, or open-ended questions; interviews; homework; projects; journals; essays; portfolios; presentations; and dramatizations. These techniques are appropriate for students in whole-class settings, small groups, or individually, with the mode of assessment written, oral, or computer oriented.

Samples

The purpose of an assessment should dictate the questions asked, methods employed, and uses of resulting information. Methods for gathering information should be appropriate to the developmental level and maturity of the students.

The following samples are excerpted and adapted from the NCTM *Curriculum and Evaluation Standards for School Mathematics*.

Sample 1

Individuals' pulse rates vary. What is the normal pulse rate for students in your class? Consider various characteristics and conditions (such as exercise). How do they relate to pulse rate?

Evaluation should focus on the reasonableness of students' questions, representations of data, verification of results, and generalizations. This exercise is appropriate for group work and can extend over several days. A score can be given to each group, and specific segments of work can be scored separately. Calculators and computers should be employed.

Sample 2

Three of five doctors interviewed recommend Dinosaur vitamins. Write a question to go with this statement to make a problem and solve it.

This problem is appropriate for large-group discussion. Assessment should focus on the choice of questions which must be logically connected to the statement. Solving one's own problem gives an indication of ability to solve and encourages the formulation of reasonable questions.

Sample 3

Seven students have test scores of 57, 96, 73, 86, 92, 75, and 89. Find the average score. How much is the average score increased if each student's score is increased by

a. 5 points?
b. x points?

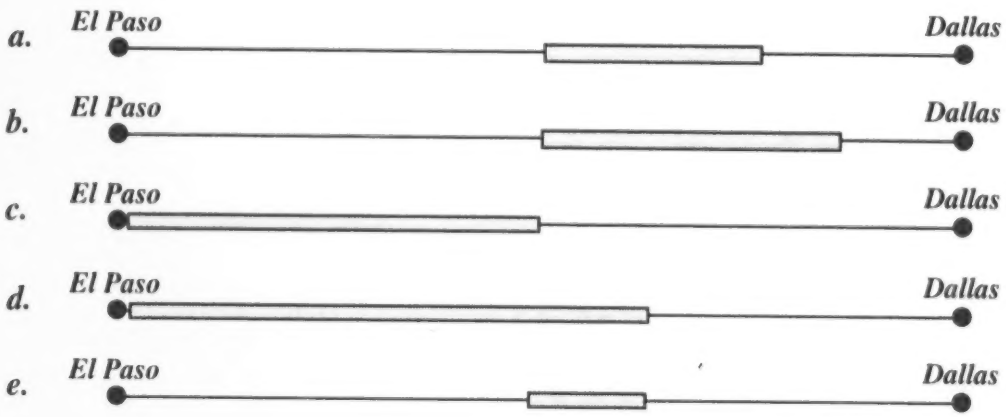
Write a statement about how much the average score is increased if each individual score is increased by x points. Convince another student that the statement is true.

Students can work in small groups. A computer is useful for investigating other situations or to consider the scores of an entire class. This sample focuses on students' ability to generalize from specific cases.

Sample 4

On a trip from El Paso to Dallas, Sara fell asleep after half the trip. When she awoke, she still had to travel half the distance she traveled while sleeping. For what part of the entire trip had she been asleep? Assuming that the shaded part in each diagram shows when Sara was asleep, which diagram best depicts the answer?

This sample assesses whether students can interpret the problem and identify the correct representation of the solution. Fractions must be considered in relation to different units. Students must think of the trip as a whole and then consider the portion of the total trip during which Sara was asleep.



Sample 5

Find the least common multiple of the following numbers:

a. 7 and 49
b. 8 and 9
c. 1 and 5
d. 5 and 6 and 20

Assessment should focus on whether students can arrive at the correct answer with facility. Note that in selection "a" one number is a multiple of the other; in selection "b" the numbers are relatively prime; in answer "c" one number is 1; and "d" might require a modification of procedures previously used.

Test-Taking Strategies

1. If the score is based on the number of questions you answer correctly, you should answer every question (even if you have to guess). Do this before going to the next problem.
2. You should put a light mark next to questions that you may be able to work but that will require more time. If time permits, you can then try to work these. Erase the light marks before turning in your answer sheet.
3. Make sure you scan all of the answer choices. Some of the choices are those that are obtained when a common error is made.
4. Sometimes the answer choices may give you an idea as to how to work a problem.
5. If you do not know how to solve a problem using standard methods you may try working it backwards--that is trying each answer and seeing which one works. As your first test, pick a middle-sized answer and plug it in. Is it to little? If so, then you need answers with bigger numbers. If this middle-sized answer is too big, then test only the answer choices with smaller numbers.
6. Estimating the result of a certain calculation can reduce the amount of time needed to obtain a solution. If the answers are spread apart from each other, estimating is easy. If they are close to each other, you may not be able to estimate effectively.
7. Sometimes answer choices can be eliminated because you can see without computation that they cannot possibly be right.
8. Make sure that the question you answer is the question that was asked.
9. When first approaching a word problem, put down your pencil, read the problem over quickly to get the general idea, identify the desired unkown, and scan quickly over the answer choices.
10. Some problems contain too much information. Weed out that which is unnecessary as you set up equations, and don't worry if you do not use all the information given.
11. Remember that an answer can be represented in more than one way. If you're fairly sure you did a problem correctly, check to see if the answer you got is the same as one of the answer choices given for the problem but given in a different form.
12. When you can eliminate one answer choice or more, try to make an educated guess and record an answer.
13. Make full use of the time alloted. If you have attempted all of the problems and have some time left, take the time to check your work.
14. If you are allowed to use scratch paper, use it. Do not recopy a problem from a test to the scratch paper if it is unnecessary.
15. Avoid spending too much time on any one question.
16. Make sure that the answer sheet is filled correctly. Be careful when erasing; take the time necessary to ensure that answers to problems worked are placed correctly on the answer sheet.

Grading

Different purposes are served by assessing students' knowledge in the mathematics classroom. One of these purposes is the grading of students. Teachers are faced with a number of decisions that they must make concerning timing, form, rigorousness, and usability of assessment. Any single form of assessment is too limiting to describe fully a student's mathematical knowledge. Thus a variety of methods over time would best evaluate the different aspects of mathematical knowledge.

Standard 1 in the Evaluation Section of the *Standards* is titled "Alignment." Standard 1 calls for the methods and tasks for assessing students' learning to be aligned with the curriculum's:

1. Goals, objectives, and mathematical content;
2. Relative emphases given to various topics and processes and their relationships;
3. Instructional approaches and activities, including the use of calculators, computers, and manipulatives.

Teachers might begin changes in classroom assessment following five steps suggested by Lester and Kroll (1991).

Step 1: Start small.

Step 2: Incorporate assessment into the class routine.

Step 3: Set up an easy and efficient record-keeping system.

Step 4: Establish an assessment plan.

Step 5: Personalize the assessment plan.

In assessing students, teachers need to remember that giving a correct answer does not mean a student knows a concept. Students may give a correct solution for the wrong reason. Thus information on reasoning must come from open-ended questions and verbal interactions. Effective assessment of critical thinking depends to a great extent on how well teachers facilitate the communication of evidence of students' understandings, critical thinking, and reasoning.

Finally, portfolios have become increasingly used in mathematics assessment. The article by Jean Stenmark in Appendix, Grade 8, "Math Portfolios: A New Form of Assessment," discusses the purpose and uses of portfolios to assess student learning. Included in a portfolio might be: written descriptions of the results of mathematical investigations; extended analyses of problem situations; descriptions and diagrams; statistics and graphics; reports; responses to open-ended questions or homework; group reports; copies of awards; photographs of projects; and other ideas. Dates should be placed on all entries. Portfolios can provide essential evidence of performance and ability beyond factual knowledge.

Homework

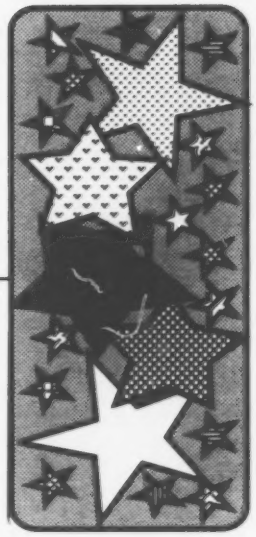
Assignments may be given for completion in class or at home. Two purposes of assignments might be to train students in meeting responsibility and to provide practice after a skill-oriented lesson. The assignments may be scored by the students themselves or by the teacher. Starting assignments during class time provides students the opportunity to enlist needed help from the teacher or peers. Homework provides an opportunity for parental involvement.

David Johnson in *Every Minute Counts* offers some possible starts to a class period that are more effective than roll call. For example, as soon as students enter the classroom, they may take out paper and pencil and solve a typical problem chosen by the teacher from the previous night's assignment. If only a few students are able to solve the exercise, reteaching is in order. Also, answers to homework assignments may be written on transparencies before students enter for checking work at the start of class. A walk around the class to view scores on each page indicates how well each student is mastering objectives. Collecting homework immediately after checking inhibits learning from mistakes. If students are not provided opportunities to see and correct errors, there is a good chance they will make the same mistakes again.

A few basic guidelines for assigning homework suggested by David Johnson are listed below.

- Know what is being assigned.
- Know what the students can do.
- Know the importance of timing. (Don't give homework too early in the class or just as the students are dismissed.)
- Make sure the students understand the assignment.
- Be careful in assigning reading in the mathematics textbook.
- Be positive about homework.
- Don't spend precious planning time correcting papers.
- Think about valuing homework scores in grading.

Appendix



The 12 Most Important Things You Can Do to Be a Better Math Teacher

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The 12 Most Important Things You Can Do to Be a Better Math Teacher
by Marilyn Burns
1993
Instructor, vol. 102, number 8
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Not too long ago, teachers saw the main goal of math instruction as helping children become proficient in paper-and-pencil computation. Today, mathematics instruction is less about teaching basic computation and more about helping students become flexible thinkers who are comfortable with all areas of mathematics and are able to apply mathematical ideas and skills to a range of problem-solving situations.

Making these goals a daily part of math instruction may require a shift in the way you think about mathematics and your role in teaching it. And so I offer the following suggestions and examples from actual classroom lessons to help you rethink your teaching practices.

1
Set the following expectation for your students: Do only what makes sense to you.

Too often, students see math as a collection of steps and tricks that they must learn. And this misconception leads to common recurring errors in arithmetic calculations--when subtracting, students will subtract the smaller from the larger rather than borrowing; when dividing, they'll omit a zero and wind up with an answer that is ten times too small; and when combining fractions, they'll erroneously add both the numerators and denominators. In all instances, students arrive at answers that make no sense, and they neither notice nor care.

There is no place for children doing mathematics by rote. Students must be able to explain the purpose for what they're doing, the logic of their procedures, and the reasonableness of their solutions.

2
Have your students explain their reasoning in all instances.

It is insufficient and shortsighted to rely on quick, right answers as indications of students' mathematical power. During math lessons, probe children's thinking when they respond. Ask: Why do you think that? Why does that make sense? Convince us. Prove it. Does anyone have a different way to think about the problem? Does anyone have another explanation?

When children are asked to explain their thinking, they are forced to organize their ideas. They have the opportunity to develop, cement, and extend their understanding. Teachers are accustomed to asking students to explain their thinking when their responses are incorrect. It's important, however, to ask children to explain their reasoning at all times, even when their answers are correct.

3
Encourage children to talk with one another during math class.

Communication is essential for learning. Having students work quietly--and by themselves--limits their opportunity for learning. Interaction maximizes children's opportunities to talk about their ideas, get feedback for their thinking, and hear other points of view. The challenge today is to have students engage in dialogue and work together to solve problems and bring meaning to mathematical ideas. Students can learn from one another, as well as from teachers.

4
Make writing an integral part of math learning.

Communication in math class should include writing as well as talking. In his book, *Writing to Learn*, William Zinsser states: "Writing is how we think our way into a subject and make it our own." When children write in math class, they have to revisit their thinking and reflect on their ideas. And student writing gives teachers a way to assess how their students are thinking and what they understand. Writing in math class best extends from children's talking. When small-group interaction or a whole-class discussion precedes a writing assignment, students have a chance to formulate their ideas before they're expected to write. It's also helpful to write prompts on the board for the students to use to get started. For example: I think the answer is____. I think this because____.

5
Embed math activities in contexts.

When connected to situations, mathematics comes alive. Contexts give students access to otherwise abstract ideas. They stimulate student interest and provide a purpose for learning mathematics. Contexts can draw on real-life examples, or they can be created from imaginary situations. Many children's books, for example, offer starting points for classroom mathematics lessons.

6
Use manipulative materials whenever possible.

Manipulative materials provide a concrete context for thinking about mathematics. They give children hands-on experiences for grabbing onto mathematics ideas, turning them around, and viewing them in different ways. Manipulative materials can serve in several ways--to introduce concepts, to pose problems, and to serve as tools to figure out solutions. It's important that manipulatives are not relegated only to young children but that they are made available to students in every grade.

7
Bring the quality and richness often apparent in students' writing and art into their math work.

Typically in elementary classrooms, children's rich, varied, and creative art and writing fill the walls, while the math work that is posted commonly consists of arithmetic worksheets or progress charts that track students' learning of basic facts. And in language arts and art lessons, children's imaginative ideas are invited and applauded; unique and unexpected results are common. In math lessons, students often learn and practice specific prescribed methods; consistency among student papers is desired.

Find ways to make math lessons and assignments as intriguing, rich, and motivating as they are in other areas of the curriculum. Give students the chance to use their creativity when thinking about mathematics. Encourage them to be inventive and trust that they will be.

8
Make calculators available to all children at all times.

Calculators are valuable tools for doing laborious mathematical computations. Owning a calculator is practically a birthright and a basic tool few adults do without. As stated in the NCTM Curriculum and Evaluation Standards for School Mathematics (page 8): "Contrary to the fears of many, the availability of calculators . . . has expanded students' capability of performing calculations."

Calculators are not a replacement for student thinking. In any problem situation, a child needs to know which buttons to push, whether the answer displayed makes sense, and what decision to make with the results. Calculators eliminate the drudge of complex calculations. They also help children solve problems they might not otherwise be able to tackle.

9
Let children push the curriculum rather than having the curriculum push the children.

Choose depth over breadth. David Hawkins has said, "You don't want to cover a subject; you want to uncover it" (*The Having of Wonderful Ideas* by Eleanor Duckworth, Teachers College Press, 1987). There are many pressures on teachers, and the school year passes very quickly. But students' understanding is key and doesn't always happen according to the schedule suggested in text materials. Just as students should do only what makes sense to them, the same is true for teachers. There is value in staying with a topic children are interested in, pushing more deeply, and taking the time for a side investigation that can extend a lesson in a different direction.

10
Keep an eye out for instructional activities that are accessible to students with different levels of interest and experience.

A wonderful quality of good children's books is that they delight adults as well. Of course, adults appreciate books for different reasons than children do, but enjoyment and learning can occur simultaneously at all levels. The same holds true for mathematics investigations. Search for activities that can engage children who have the least mathematical experience while challenging students with the most experience.

11
Remember that confusion and partial understanding are natural to the learning process.

Do not expect all children to learn everything at the same time, and do not expect all children to get the same message from every lesson. Although teachers want all their students to be successful, they rarely reach every student with any one lesson. Learning should be viewed as a long-range goal, not as a lesson objective. It's important that children do not feel deficient, hopeless, or excluded form learning mathematics. The classroom culture should reinforce the belief that errors are opportunities for learning and should support children taking risks without fear of failure or embarrassment.

12
Take delight in students' thinking.

There is no one way to think about any mathematical problem. Encourage students to think in different ways. After children respond to a question (and, of course, have explained their thinking!), ask: Does anyone have a different idea. Keep asking until all children who volunteer have offered their ideas. By encouraging participation, you'll not only learn more about individual children's thinking, but you'll also send students the message that there is more than one way to look at any problem or situation.

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COMPLIANCE STATEMENT

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